Annex xxx
General guidelines for the assessment of uncertainty in solar collector efficiency testing

1. Introduction

Testing laboratories, in the framework of their accreditation or of application of product certification schemes, are often invited to provide a statement of uncertainty in test results. Even though the assessment of uncertainty concerns every quantitative result, the most interesting quantity for the potential users of solar thermal collectors is the one of the energy efficiency, as this quantity essentially influences the cost-benefit relation of the required investment.

Moreover, the quantification of the various error sources and the analysis of the uncertainty budget are considered notably significant for the testing laboratories, to the degree that the uncertainty budget analysis can help towards the optimization of the testing procedure, the effective planning of testing equipment and the improvement of the testing results quality.

The aim of this annex is to provide a general guidance for the assessment of uncertainty in the result of solar collector testing performed according to the present standard. The need for a well defined methodology for the assessment of uncertainty in collector efficiency testing results arises due to the peculiarities of the related calculations. More specifically, the final result is not derived by a single direct measurement, but it is the outcome of the combination of a large number of primary measurements supported by intermediate calculations, on a procedure consisting of multiple stages.

It is important to note that the proposed methodology is one amongst the possible approaches for the assessment of uncertainty; other approaches can also be implemented, given that they are compatible with the up-to-date metrological concepts for the estimation of metrological uncertainty (BIPM et al., 2008). It lies upon each Laboratory to choose and to implement a scientifically valid approach for the determination of uncertainties, according to the recommendations of the accreditation bodies, where appropriate. For a more detailed review of the different aspects of determination of uncertainties in solar collector testing see also (Mathioulakis *et al*., 1999; Sabatelli *et al*., 2002; Müller-Schöll and Frei, 2000).

2. The measurement model

The basic target of solar collector efficiency testing is the determination of the coefficients of the characteristic equation of the solar collector, through the measurement of the efficiency in certain conditions. More specifically, it is assumed that the energy performance of the collector can be described by a M-parameter single node, steady state or quasi-dynamic model (measurement model):

$$y = c_1 x_1 + c_2 x_2 + \ldots + c_M x_M \qquad (1)$$

where:

y is a dependent variable, typically a quantity related to the collector efficiency, the values of which are determined experimentally through testing.

$x_1,\ x_2,\ldots,x_M$ are independent variables, the values of which are also determined experimentally through testing.

$c_1, c_2, \ldots, c_M$ are characteristic constants of the collector.

The aim of the test is the determination of the values of the characteristic coefficients $c_1, c_2, \ldots, c_M$ in a manner according to which the experimental values of y and $x_1, x_2, \ldots, x_M$ could fit as good as possible with equation 1 (best fit).

In the case of the steady state model: $M=3$, $y=n$, $c_1=\eta_0$, $c_2=U_1$, $c_3=U_2$, $x_1=1$, $x_2=(T_m-T_a)/G$ and $x_3=(T_m-T_a)^2/G$.

In the case of the quasi-dynamic model, in its simplest form, the respective parameters are: $M=6$, $y=\dot{Q}/A$, $c_1=\eta_{0,b \cdot en}$, $c_2=\eta_{0,b \cdot en} b_0$, $c_3=\eta_{0,b \cdot en} K_{\theta d}$, $c_4=c_1$, $c_5=c_2$, $c_6=c_5$, $x_1=G_b$, $x_2=-G_b\left(\frac{1}{\cos(\theta)}-1\right)$, $x_3=G_d$, $x_4=-(t_m-t_a)$, $x_5=-(t_m-t_a)^2$, $x_6=-\frac{dt_m}{dt}$. Similar is the treatment of quantities which are involved in the other forms of the quasi-dynamic model, namely when the influence of the wind velocity or of long-wave thermal radiation is considered.

During the experimental phase, the output energy of the collector and the involved climatic quantities (incident solar radiation, ambient temperature, wind velocity, etc.), are measured in J steady-state or quasi-dynamic state points, depending on the model used. From these primary measurements the values of parameters $y, x_1, x_2, \ldots, x_M$ are derived for each point of observation j, $j=1 \ldots J$. Generally, the experimental procedure of the testing leads to a formation of a group of J observations which comprise, for each one of the J testing points, the values of $y, x_1, x_2, \ldots, x_M$.

3. Uncertainties associated with experimental data

For the determination of uncertainties, it is essential to calculate the respective combined standard uncertainties u_{y_j}, $u_{x_{j,1}}$, …, $u_{x_{j,M}}$ of the dependent variable, as well as of the independent ones, in each observation point. It should be noted that in practice these uncertainties are almost never constant and same for all points, but each testing point has its own standard deviation.

The uncertainty in the quantities correlated through equation 1 is not directly known, but it can be calculated according to the metrological characteristics of the measurement devices. For the calculation of the standard deviation (squared standard uncertainty) in each point j, the following general rules can be applied (BIPM et al., 2008).

I. Standard uncertainties in experimental data are determined by taking into account Type A and Type B uncertainties. According to the recommendation of ISO GUM (BIPM et al., 2008), the former are the uncertainties determined by statistical means while the latter are determined by other means.

II. The uncertainty u_s associated with a measurement s of a quantity S is the result of a combination of the Type B uncertainty $u_{B,s}$, which is a characteristic feature of the calibration setup, and of the Type A uncertainty $u_{A,s}$, which represents fluctuation during sampling of data. If there is more than one independent source of uncertainty (Type B or Type A) u_i, $i=1,\ldots,I$, the final uncertainty is calculated according to the general law of uncertainties combination:

$$u_s = \left(\sum_{i=1}^{I} u_i^2\right)^{1/2} \tag{2}$$

III. Type B uncertainty $u_{B,s}$ derives from a combination of uncertainties over the whole measurement chain, taking into account all available information, such as sensor uncertainty, data logger uncertainty, uncertainty resulting from the possible differences between the measured values perceived by the measuring device. Relevant information should be obtained from calibration certificates or other technical data related to the devices used.

IV. By nature, Type A uncertainties depend on the specific conditions of measurement and they account for the fluctuations in the measured quantities during the measurement. Type A uncertainty $u_{A,s}$ derives from the statistical analysis of experimental data. In some cases (for example in the case of the steady-state model), the best estimate s of S is the arithmetic means of the N repeated observations s_n ($n=1...N$) and its Type A uncertainty is the standard deviations of the mean:

$$s = \frac{\sum_{n=1}^{N} s_n}{I}, \text{ and } u_{A,s} = \left(\frac{\sum_{n=1}^{N} (s_n - s)^2}{N(N-1)} \right)^{1/2} \tag{3}$$

In cases where no arithmetic mean of the repetitive measurements is used, as in the case of the quasi-dynamic model, uncertainty $u_{A,s}$ is equal to zero.

V. The term *combined standard uncertainty* means the standard uncertainty in a result when that result is obtained from the values of a number of other quantities. In most cases a measured Y is determined indirectly from P other directly measured quantities $X_1, X_2,...,X_P$ through a functional relationship $Y=f(X_1, X_2, ...X_P)$. The standard uncertainty in the estimate y of Y is given by the *law of error propagation,* as a function of the estimates $x_1, x_2, ...,x_P$ of $X_1, X_2,...,X_P$, taking also into account the respective standard uncertainties $u_{x1}, u_{x2}, ...u_{xp}$:

$$u_y = \left(\sum_{i=1}^{P} \left(\frac{\partial f}{\partial x_i} u_{xi} \right)^2 + 2 \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \operatorname{cov}(x_i, x_j) \right)^{1/2} \tag{4}$$

In cases the estimates $x_1, x_2, ...,x_P$ can be considered as independent one to the other, the relation (4) is simplified accordingly:

$$u_y = \sqrt{\sum_{i=1}^{P} \left(\frac{\partial f}{\partial x_i} u_{xi} \right)^2} \tag{5}$$

The uncertainty in the quantities correlated through equation 1 is not directly known, but it can be calculated according to the metrological characteristics of the measurement devices. For this indirect calculation, the law of error propagation is implemented (BIPM et al., 2008).

An example of such indirect determination in the case of solar collector efficiency testing is the determination of instantaneous efficiency η, which derives from the values of global solar irradiance in the collector level G, fluid mass flowrate m, temperate difference ΔT, collector area A and specific heat capacity C_p:

$$n = \frac{m\,C_p(T_e - T_{in})}{A\,G} \tag{6}$$

Thus, in this case the standard uncertainty u_η in each value η of instantaneous efficiency is calculated by the combination of standard uncertainties in the values of the primary measured quantities, taking into account their relation to the derived quantity η. By assuming that the quantities entering the second part of relation 6 are not correlated, and the value of C_p is known with negligible uncertainty, the standard uncertainty in the values of n can be estimated by the relation:

$$u_n = \sqrt{\left(\frac{\partial n}{\partial m}u_m\right)^2 + \left(\frac{\partial n}{\partial T_e}u_{T_e}\right)^2 + \left(\frac{\partial n}{\partial T_{in}}u_{T_{in}}\right)^2 + \left(\frac{\partial G}{\partial G}u_G\right)^2 + \left(\frac{\partial G}{\partial A}u_A\right)^2} = $$
$$= n\sqrt{\left(\frac{u_m}{m}\right)^2 + \left(\frac{u_{T_e}}{T_e - T_{in}}\right)^2 + \left(\frac{u_{T_{in}}}{T_e - T_{in}}\right)^2 + \left(\frac{u_G}{G}\right)^2 + \left(\frac{u_A}{A}\right)^2} \tag{7}$$

The information related to the uncertainty characterizing the primary measured quantities has to be derived by information coming from the calibration of sensors used in practice.

4. Fitting and uncertainties in efficiency testing results

Following the completion of the test, the elaboration of the primary experimental data leads to J testing points, namely J sets of values of the dependent variable y and the M independent variables $x_1, x_2, \ldots, x_M$, which can be written in matrix form:

$$Y_e = \begin{bmatrix} y_i \\ \cdot \\ \cdot \\ \cdot \\ y_J \end{bmatrix},\; X_e = \begin{bmatrix} x_{1,1} & \cdot & \cdot & \cdot & x_{1,M} \\ \cdot & \cdot & & & \cdot \\ \cdot & & x_{j,m} & & \cdot \\ \cdot & & & \cdot & \cdot \\ x_{J,1} & \cdot & \cdot & \cdot & x_{J,M} \end{bmatrix} \tag{8}$$

A least square fitting of the model equation is performed, in order to determine the values of coefficients $c_1, c_2, \ldots, c_M$ for which the model of equation 1 represents the series of J observations with the greatest accuracy. However, very often, in order to be in compliance with the requirements of accreditation and certification, not only the coefficients, but also their variances and covariances are required for uncertainty analysis of results produced by any further use of the fitted model.

The deviations of the model from the real data can be attributed to experimental errors but also to model weaknesses. In any case, the basic working hypothesis considers the model to be suitable for the description of the related to experimental observations phenomena.

The basic methodology is almost always the same (Press et al., 1996): a *figure-of-merit function* is selected, to give an indication of the difference between the real data and the model. After this, the model parameters are selected so that the value of this function is minimized.

The most commonly used method for the fitting is this of the ordinary least squares (OLS), being also easy to implement. Ordinary least squares technique is based on a set of hypotheses that are not always fulfilled, mainly referring to the absence of errors on the values of the independent variables and, often, to the homoscedasticity of the errors associated with the values of dependent variable. However, in practice, measured data are always subject to some uncertainty. Thus, through the procedure of best fit determination, as in the case treated here, a method is required for fitting the linear model to data with uncertainties in both dependent and independent coordinates.

In the case where only the values of the dependent variables are precisely known, the figure-of-merit function to be minimized presents the following form:

$$\chi^2 = (Y - Y_e)^T (u_Y^2)^{-1} (Y - Y_e) \tag{9}$$

Where u_Y^2 is the covariance (or uncertainty) matrix associated with the vector *Y*. Assuming that the covariance matrix u_Y^2 is diagonal, that is to say that the measurements $Y_1, \ldots, Y_J$ are independent one to the other, the relation 9 is written:

$$\chi^2 = \sum_{j=1}^{J} \left(\frac{y_j - (c_1 x_{j,1} + c_2 x_{j,2} + \ldots + c_M x_{j,M})}{u_{k_j}} \right)^2 \tag{10}$$

For the case the uncertainties characterizing the values $x_{j,m}$ of the independent variables can not be considered as negligible, the figure-of-merit function to be minimized becomes (Lira, 2000):

$$\chi^2 = (X - X_e)^T (u_X^2)^{-1} (X - X_e) + (Y - Y_e)^T (u_Y^2)^{-1} (Y - Y_e) \tag{11}$$

where u_X^2 is the covariance (or uncertainty) matrix associated with the matrix *X*.

The derivation of equation 11 for the finding of its minimum value is quite complicated due to the non linear character it presents. For the needs of usual metrological applications, various approximate methods have been proposed, each one presenting different degree of difficulty. Within the context of the present investigation, the so-called "effective variance" approach is adopted, the implementation of which is justified in cases where the input uncertainty matrices can be considered as diagonal, as it is valid for the case concerning this work.

According to this approach, the overall variance in each observation point results through the "transferring" of the uncertainties in X to those in Y and treating X as exactly known quantities (Lira, I.; Cecchi, 1991; Press *et al.*, 1996):

$$u_j^2 = \mathrm{var}\left(y_j - \left(c_1 x_{j,1} + c_2 x_{j,2} + \ldots + c_M x_{j,M}\right)\right) = y_j^2 + c_1^2 u_{xj,1}^2 + c_M^2 u_{xj,1}^2 + \ldots + c_M^2 u_{xj,M}^2 \qquad (12)$$

In this case, the normal equation of the least square problem can be written:

$$\left(K^T \cdot K\right) \cdot C = K^T \cdot L \qquad (13)$$

where C is a vector whose elements are the fitted coefficients, K is a matrix whose JxM components $k_{j,m}$ are constructed from M basic functions evaluated at the J experimental values of $x_1, \ldots, x_M$ weighted by the uncertainty u_j , and L is a vector of length J whose components l_j are constructed from values of y_j to be fitted, weighted by the uncertainty u_j:

$$k_{j,m} = \frac{x_{j,m}}{u_j}, \qquad K = \begin{bmatrix} \frac{x_{1,1}}{u_1} & . & . & . & \frac{x_{1,M}}{u_1} \\ . & . & & & . \\ . & & \frac{x_{j,m}}{u_j} & & . \\ . & & & . & . \\ \frac{x_{J,1}}{u_J} & . & . & . & \frac{x_{J,M}}{u_J} \end{bmatrix} \qquad (14)$$

$$l_j = \frac{y_j}{u_j}, \qquad L = \begin{bmatrix} \frac{y_i}{u_1} \\ . \\ . \\ . \\ \frac{y_J}{u_J} \end{bmatrix} \qquad (15)$$

Given that for the calculation of variances u_j^2 the knowledge of coefficients $c_1, c_2, \ldots, c_M$ is needed, a possible solution is to use the values of coefficients calculated by ordinary least squares fitting as the initial values. These initial values can be used in equation 12 for the calculation of u_j^2, $J=1 \ldots J$ and the formation of matrix K and of vector L.

The solution of equation 13 gives the new values of coefficients $c_1, c_2, \ldots, c_M$, which however are not expected to differ noticeably from those calculated by standard least squares fitting and used as initial values for the calculation of u_j^2:

$$C = \left(K^T K\right)^{-1}\left(K^T L\right) \quad (16)$$

Moreover, Z=INV(K^T•K) is the uncertainty matrix whose diagonal elements $z_{k,k}$ are the squared uncertainties (variances) and the off-diagonal elements $z_{k,l}= z_{l,k}$, $k\neq l$ are the covariance between fitted coefficients:

$$u_{c_m} = \sqrt{z_{mm}}\,,\ m=1,\dots,M \quad (17)$$

$$Cov(c_k,c_l)= z_{k,l}= z_{l,k},\ k=1,\dots,M \text{ and } l=1,\dots,M \text{ and } k\neq l \quad (18)$$

It should be noted that the knowledge of covariance between the fitted coefficients is necessary if one wishes to calculate, in a next stage, the uncertainty u_x in the predicted values of x using equations 1 and the law of error propagation (equation 4).

Equation 16 can be solved by a standard numerical method, for example, by Gauss-Jordan elimination. It is also possible to use matrix manipulation functions of commonly used spreadsheet software.

What most concerns future users of the collectors is the uncertainty characterizing the values of the collector efficiency, when this is calculated for given values of operation conditions (irradiance, angle of incidence, ambient temperature, inlet water temperature etc). The calculation of the expected efficiency $y^{/}$ can be easily done by entering in equation 1 the fitted coefficients $C = \begin{bmatrix} c_1 & . & . & . & c_M \end{bmatrix}^T$ and the operation conditions $X^{/} = \begin{bmatrix} x^{/}_1 & . & . & . & x^{/}_M \end{bmatrix}$:

$$y = \sum_{m=1}^{M} c_m x^{/}_m \quad \text{or in matrix notation } y^{/} = X^{/} \cdot C \quad (18)$$

The uncertainty in the predicted values of y can be calculated by applying the law of error propagation (equation 4), taking into account both variances and covariances of the fitted coefficient and considering the operation conditions $X^{/}$ to be known without uncertainties:

$$u_{y^{/}} = \sqrt{\sum_{m=1}^{M}\left(x^{/}_m u_{C_m}\right)^2 + 2\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} x^{/}_i x^{/}_j \operatorname{cov}(c_i, c_j)} \quad (19)$$

or in matrix notation:

$$u_{y^{/}} = \sqrt{X^{/} \cdot Z \cdot \left(X^{/}\right)^T} \quad (20)$$