

Project Final report

Project title: Estimation of uncertainty of determined collector and system performance (UNCERT)

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Assessment of uncertainty in efficiency testing of solar thermal collectors and solar thermal systems

1. Introduction

Testing laboratories, in the framework of their accreditation or of application of product certification schemes, are often invited to provide a statement of uncertainty in test results. Even though the assessment of uncertainty concerns every quantitative result, the most interesting quantity for the potential users of solar thermal energy installations is the one of the energy efficiency, as this quantity essentially influences the cost-benefit relation of the required investment.

Moreover, the quantification of the various error sources and the analysis of the uncertainty budget are considered notably significant for the testing laboratories, to the degree that the uncertainty budget analysis can help towards the optimization of the testing procedure, the effective planning of testing equipment and the improvement of the testing results quality, on a wider context.

The aim of this analysis is to provide a general guidance for the assessment of uncertainty characterizing the results of solar thermal collectors and systems testing, performed according to the standards EN 12975-2 and EN 12976-2 respectively. The need for a well defined methodology for the assessment of uncertainty in efficiency testing results arises due to the peculiarities of the related calculations. More specifically, the final result is not derived by a single direct measurement, but it is the outcome of the combination of a large number of primary measurements supported by intermediate calculations, on a procedure consisting of multiple stages.

It is important to note that the proposed methodology is one amongst the possible approaches for the assessment of uncertainty; other approaches can also be implemented, given that they are compatible with the up-to-date metrological concepts for the estimation of metrological uncertainty (BIPM et al., 2008a). It lies upon each Laboratory to choose and to implement a scientifically valid approach for the determination of uncertainties, according to the recommendations of the accreditation bodies, where appropriate.

It should also be noted that the uncertainty associated with a quantitative result is not a property of the measured quantity itself, but it characterizes the quality of information that the metrologist or the laboratory have achieved to acquire for the specific quantity. On this point of view, the uncertainty value strongly depends on the metrological performance of the experimental equipment which is used, as well as on the procedures implemented for the specific data collection and elaboration.

This report is organized as follows. In section 2 specific aspects related to the assessment of uncertainties in general, as well as to the calculation of uncertainties in the case of solar thermal products performance testing in particular, are examined. Section 3 focuses on the case of solar thermal collectors testing, and it includes, apart from the description of the proposed methodology, an example of implementation on real testing data. Section 4 concerns the solar thermal systems testing, making reference to both methods which are accepted within the framework of European accreditation.

2. General rules for the calculation of uncertainties in efficiency testing

2.1. Testing methods of solar thermal products

The uncertainties calculations which are discussed in this study concern the testing of performance of solar thermal collectors according to the Standard *EN 12975-2: Thermal solar systems and components - Solar collectors - Part 2: Test methods*, and the testing for thermal performance of solar thermal systems according to the Standard *EN 12976-2: Thermal solar systems and components - Factory made systems - Part 2: Test methods* (CEN, 2006a; CEN, 2006b).

A common characteristic of the solar collectors and systems efficiency testing is the fact that the relevant procedures do not represent direct measurements themselves. On the contrary, the results come up on the basis of a rather complicated procedure, the basic elements of which are the following:

- According to the Standards, the energy behavior of a collector or a system can be sufficiently described by a specific model which relates the energy efficiency (output) with a series of parameters (mainly climatic ones) which influence the operation of the device (inputs). It is noted though, that in the cases of both collectors and systems there have been proposed more than one models, presenting significant differences the one to the other. The aim of the testing is the determination of the characteristic (constant) coefficients of the used model. Provided that the values of these coefficients are known, the model can be used for the prediction of the expected energy benefit on the assumed conditions.
- The models that are used are not completely accurate, to the degree that their development is based on the adoption of specific simplifications. The degree of accuracy according to which each of the assumed models describes the actual energy behaviour of each product (collector or system) depends, at first place, on the quality of the model. Moreover, it depends on the proper implementation of the anticipated testing procedures, as well as on the testing product itself.
- Even though each method presents certain individual characteristics, the implementation of both methods includes four basic phases:
 - i. On the first phase, the experimental measurements are implemented, according to a clearly described testing protocol. These measurements concern, on one hand the parameters which influence the energy behavior of the device under test, such as climatic data, auxiliary heating, etc., and on the other hand quantities which can be directly related with the energy performance of the device.
 - ii. On the second phase, elaboration of the initial experimental data takes place, in order to produce experimental values for the inputs and outputs of the model, on a way which can cover as realistic as possible the anticipated operation range of the collector or system.
 - iii. On the third phase, the determination of the model coefficients is attained, on a way which can ensure the best possible correlation of the output(s) and inputs. The methodology adopted is that of the least square regression, even though there are specific peculiarities characterizing each testing procedure.

- iv. Finally, the fitted model is used for the assessment of the expected energy output under the specific operation conditions of the product.

From the above, it is obvious that the calculation of uncertainties mostly concerns the final efficiency indicators which are provided to the client of the laboratory. It can be also clear that this calculation should count in the uncertainties associated with potential errors of the measuring equipment, as well as these attributed to the weakness of the energy model to accurately describe the actual behavior of the device under test.

It should be noted that the uncertainty calculated by the testing laboratory, concerns the specific product which has been tested and not the total production of the manufacturer. It is impossible for the final uncertainty budget to include the uncertainty component which is associated with potential differences from product to product of the same type. These differences are related with the repeatability of the production procedures of each enterprise, as well as with the keeping of constant properties for the raw materials used. The assessment of this source of uncertainty could only be feasible on the basis of a statistical study; this statistical study should consider a sampling protocol of the products which would count in all sources of variability of the specific productive process.

2.2. General principles for the estimation of uncertainty

The problem of performing an uncertainty budget is actually quite straightforward in most cases, especially when direct measurements are concerned. There are cases though, where the estimation of uncertainty characterizing the quantitative results of the tests can be proven to be particularly complex. As a consequence, different and in some cases conflicting uncertainty evaluation procedures can be proposed.

Nevertheless, in order to receive general acceptance, uncertainty analysis has to take into account some general principles which have been agreed on international level. It is for this reason that, by recent years, the international metrological community has focused on the elaboration of a common framework of principles for the calculation of uncertainties. The outcome was the *Guide to the Expression of Uncertainty in Measurement*, known popularly as the **GUM**, which is intended to orient metrologists into the complexities of the evaluation of uncertainty (BIPM, 2008a).

From this point of view, it would not be worthless to include in the text some essential elements, on the basis of which the estimation of the uncertainty characterizing a specific quantitative result is performed:

- I. Standard uncertainties in experimental data are determined by taking into account Type A and Type B uncertainties. According to the recommendation of ISO GUM (BIPM et al., 2008a), the former are the uncertainties determined by statistical means while the latter are determined by other means.
- II. The uncertainty u_s associated with a measurement s of a quantity S is the result of a combination of the Type B uncertainty $u_{B,s}$, which is a characteristic feature of the calibration setup, and of the Type A uncertainty $u_{A,s}$, which represents fluctuation during sampling of data. If there is more than one independent source of uncertainty (Type B or Type A) u_i , $i=1, \dots, I$, the final uncertainty is calculated according to the general law of uncertainties combination:

$$u_s = \left(\sum_{i=1}^I u_i^2 \right)^{1/2} \quad (2.1)$$

III. Type B uncertainty $u_{B,s}$ derives from a combination of uncertainties over the whole measurement chain, taking into account all available information, such as sensor uncertainty, data logger uncertainty, uncertainty resulting from the possible differences between the measured values perceived by the measuring device. Relevant information should be obtained from calibration certificates or other technical data related to the devices used.

IV. By nature, Type A uncertainties depend on the specific conditions of measurement and they account for the fluctuations in the measured quantities during the measurement. Type A uncertainty $u_{A,s}$ derives from the statistical analysis of experimental data. In some cases (for example in the case of the steady-state model), the best estimate s of S is the arithmetic means of the N repeated observations s_n ($n=1\dots N$) and its Type A uncertainty is the standard deviations of the mean:

$$s = \frac{\sum_{n=1}^N s_n}{N}, \text{ and } u_{A,s} = \left(\frac{\sum_{n=1}^N (s_n - s)^2}{N(N-1)} \right)^{1/2} \quad (2.2)$$

In cases where no arithmetic mean of the repetitive measurements is used, as in the case of the quasi-dynamic model, uncertainty $u_{A,s}$ is equal to zero.

V. The term *combined standard uncertainty* means the standard uncertainty in a result when that result is obtained from the values of a number of other quantities. In most cases a measured Y is determined indirectly from P other directly measured quantities X_1, X_2, \dots, X_P through a functional relationship $Y=f(X_1, X_2, \dots, X_P)$. The standard uncertainty in the estimate y of Y is given by the *law of error propagation*, as a function of the estimates x_1, x_2, \dots, x_P of X_1, X_2, \dots, X_P , taking also into account the respective standard uncertainties $u_{x1}, u_{x2}, \dots, u_{xp}$:

$$u_y = \left(\sum_{i=1}^P \left(\frac{\partial f}{\partial x_i} u_{x_i} \right)^2 + 2 \sum_{i=1}^{P-1} \sum_{j=i+1}^P \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \text{cov}(x_i, x_j) \right)^{1/2} \quad (2.3)$$

In cases the estimates x_1, x_2, \dots, x_P can be considered as independent one to the other, the relation (2.3) is simplified accordingly:

$$u_y = \sqrt{\sum_{i=1}^P \left(\frac{\partial f}{\partial x_i} u_{x_i} \right)^2} \quad (2.4)$$

According to the Law of Error Propagation, the information transferred through each experimental observation, can be summarized on the value of the measured quantity (measurand), and the standard uncertainty characterizing this value. This information propagates through a first-order Taylor series expansion of the measurement model, allowing the calculation of the standard deviation (or standard uncertainty) of the final result, as described on the Guide to the Expression of Uncertainty in Measurement. The relation connecting the primary testing data to the quantity to be calculated constitutes the *measurement model*.

It has to be noted that the use of the *law of error propagation* is subject to specific constraints, mainly in cases of non linear models, non satisfaction of the Central Limit Theorem requirements, or even appearance of difficulties in the determination of the sensitivity coefficients (BIPM et al., 2008b). This final limitation is applicable in the case of solar systems, examined in this work, due to the inability to formulate specific derivable equations addressing the calculation of the expected energy output. These limitations, combined with the rapid increase of the computational capacity available to the laboratories, have favored the use of alternative approaches, referred to as the *Monte Carlo* technique, which has been the subject of the first addendum to GUM (Burhenne et al., 2010; BIPM et al., 2008b; Wubbele et al., 2008; Cox and Siebert, 2006). The selection of one or the other approach can be considered a matter of preference of the testing laboratory, provided that the metrological information used is appropriately justified.

The basic idea of this technique concerns the propagation of distribution rather than the propagation of the uncertainties. In the case of a measurement model $y = f(x_1, \dots, x_p)$ which calculates the value y of the measurand Y as a function of the experimental values x_1, \dots, x_p of the P physical quantities X_1, \dots, X_p , the implementation of the method can be summarized as follows:

- For each measurement point, the information about a given value x_i of the input quantity X_i is encoded by a specified Probability Distribution Function (PDF) g_{x_i} . This PDF can be experimentally inferred from direct repeated measurements of the input quantity or assigned to the primary input estimate on the base of the Principle of Maximum Entropy (Lira et al., 2009).

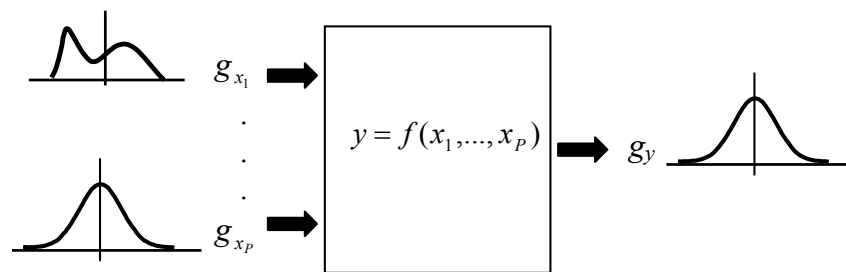


Figure 2.1: Schematic representation of the Monte-Carlo approach

- A suitable algorithm is used to generate, for each primary source value x_i , a high number sequence of N values $x_{i,j}, j=1, \dots, N$, the statistical properties of which approximate those of the respective PDF g_{x_i}

- The measurement model $y = f(x_1, \dots, x_p)$ is used N times, one time for each of the N combinations $x_{1,j}, \dots, x_{p,j}, j=1, \dots, N$. Thus, N values $y_j, j=1, \dots, N$ of the result are produced (Figure 2.1), allowing the formulation of a discrete representation g_y of the PDF for the measurand Y .
- The distribution g_y allows the calculation of the average \bar{y} and standard deviation u_y , considered the best possible estimates of the value of Y and of the associated standard uncertainty:

$$\bar{y} = \frac{1}{N} \sum_{i=1}^N y_i \quad (2.5)$$

$$u_y = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2} \quad (2.6)$$

3. Uncertainties in solar thermal collector efficiency testing

3.1. The testing method

In this section, a step-by-step methodology for the estimation of the contribution of all the uncertainty components on the basis of the EN 12975-2 test procedure is carried out, in order to determine the final uncertainty in the characteristic equation parameters and the instantaneous efficiency of a solar thermal collector.

As it has been indicated in the introduction, the procedure for the assessment of the uncertainty characterizing a quantitative result is not unique, and other approaches can also be implemented, given that they are compatible with the up-to-date metrological concepts for the estimation of metrological uncertainty. Moreover, the whole procedure has to be adjusted to the particular conditions of each laboratory. For a more detailed review of the different aspects of determination of uncertainties in solar collector testing see also (Mathioulakis et al., 1999; Müller-Schöll and Frei, 2000; Sabatelli et al., 2002).

The basic target of solar collector efficiency testing is the determination of the coefficients of the characteristic equation of the solar collector, through the measurement of the efficiency in certain conditions. More specifically, it is assumed that the energy performance of the collector can be described by a M -parameter single node, steady state or quasi-dynamic model (measurement model):

$$y = c_1 x_1 + c_2 x_2 + \dots + c_M x_M \quad (3.1)$$

where:

y is a dependent variable, typically a quantity related to the collector efficiency, the values of which are determined experimentally through testing.

x_1, x_2, \dots, x_M are independent variables, the values of which are also determined experimentally through testing.

c_1, c_2, \dots, c_M are characteristic constants of the collector.

The aim of the test is the determination of the values of the characteristic coefficients c_1, c_2, \dots, c_M in a manner according to which the experimental values of y and x_1, x_2, \dots, x_M could fit as good as possible with equation 3.1 (best fit).

In the case of the steady state model: $M=3$, $y=n$, $c_1=\eta_0$, $c_2=U_l$, $c_3=U_2$, $x_1=I$, $x_2=(T_m-T_a)/G$ and $x_3=(T_m-T_a)^2/G$.

In the case of the quasi-dynamic model, in its simplest form, the respective parameters are: $M=6$, $y=\dot{Q}/A$, $c_1=\eta_{0,ben}$, $c_2=\eta_{0,ben}b_0$, $c_3=\eta_{0,ben}K_{\theta d}$, $c_4=c_1$, $c_5=c_2$, $c_6=c_5$, $x_1=G_b$, $x_2=-G_b\left(\frac{1}{\cos(\theta)}-1\right)$, $x_3=G_d$, $x_4=-(t_m-t_a)$, $x_5=-(t_m-t_a)^2$, $x_6=-\frac{dt_m}{dt}$. Similar is the treatment of

quantities which are involved in the other forms of the quasi-dynamic model, namely when the influence of the wind velocity or of long-wave thermal radiation is considered.

During the experimental phase, the output energy of the collector and the involved climatic

quantities (incident solar radiation, ambient temperature, wind velocity, etc.), are measured in J steady-state or quasi-dynamic state points, depending on the model used. From these primary measurements the values of parameters y, x_1, x_2, \dots, x_M are derived for each point of observation $j, j=1 \dots J$. Generally, the experimental procedure of the testing leads to a formation of a group of J observations which comprise, for each one of the J testing points, the values of y, x_1, x_2, \dots, x_M .

3.2. Uncertainties associated with experimental data

For the determination of uncertainties, it is essential to calculate the respective combined standard uncertainties $u_{y_j}, u_{x_{j,1}}, \dots, u_{x_{j,M}}$ of the dependent variable, as well as of the independent ones, in each observation point. It should be noted that in practice these uncertainties are almost never constant and same for all points, but each testing point has its own standard deviation.

The uncertainty in the quantities correlated through equation 3.1 is not directly known, but it can be calculated according to the metrological characteristics of the measurement devices. For this indirect calculation, the law of error propagation is implemented, as noted in section 2.

An example of such indirect determination in the case of solar collector efficiency testing is the determination of instantaneous efficiency η , which derives from the values of global solar irradiance in the collector level G , fluid mass flowrate m , temperate difference ΔT , collector area A and specific heat capacity C_p :

$$n = \frac{m C_p (T_e - T_{in})}{A G} \quad (3.2)$$

Thus, in this case the standard uncertainty u_n in each value n of instantaneous efficiency is calculated by the combination of standard uncertainties in the values of the primary measured quantities, taking into account their relation to the derived quantity n . By assuming that the quantities entering the second part of relation 3.2 are not correlated, and the value of C_p is known with negligible uncertainty, the standard uncertainty in the values of n can be estimated by the relation:

$$\begin{aligned} u_n &= \sqrt{\left(\frac{\partial n}{\partial m} u_m\right)^2 + \left(\frac{\partial n}{\partial T_e} u_{T_e}\right)^2 + \left(\frac{\partial n}{\partial T_{in}} u_{T_{in}}\right)^2 + \left(\frac{\partial G}{\partial G} u_G\right)^2 + \left(\frac{\partial G}{\partial A} u_A\right)^2} = \\ &= n \sqrt{\left(\frac{u_m}{m}\right)^2 + \left(\frac{u_{T_e}}{T_e - T_{in}}\right)^2 + \left(\frac{u_{T_{in}}}{T_e - T_{in}}\right)^2 + \left(\frac{u_G}{G}\right)^2 + \left(\frac{u_A}{A}\right)^2} \end{aligned} \quad (3.3)$$

The information related to the uncertainty characterizing the primary measured quantities has

to be derived by information coming from the calibration of sensors used in practice.

Another example can refer to the assessment of uncertainty in the measurements of global solar irradiance G . The uncertainty in the indications of pyranometer is found by taking into account several potential sources of error: non linearity, temperature dependence of sensitivity, spectral sensitivity, cosine and azimuth response, response time and calibration errors. The value of the uncertainty component associated with each one of the error sources has to be estimated through the consideration of the respective measurement conditions as well as the characteristics of the used equipment. In the case, for example, of the steady state method, the errors associated with the spectral sensitivity, the cosine and azimuth response and the response time can be considered negligible, in view of the fact that measurements are performed under clear sky with vertical incidence and practically constant irradiance. The final value of uncertainty would be calculated through the implementation of the *law of error propagation*, namely as the square root of the quadratic sum of all the remaining uncertainties components.

3.3. Fitting and uncertainties in efficiency testing results

Following the completion of the test, the elaboration of the primary experimental data leads to J testing points, namely J sets of values of the dependent variable y and the M independent variables x_1, x_2, \dots, x_M , which can be written in matrix form:

$$Y_e = \begin{bmatrix} y_i \\ \cdot \\ \cdot \\ \cdot \\ y_J \end{bmatrix}, X_e = \begin{bmatrix} x_{1,1} & \cdot & \cdot & \cdot & x_{1,M} \\ \cdot & \cdot & & & \cdot \\ \cdot & & x_{j,m} & & \cdot \\ \cdot & & & \cdot & \cdot \\ x_{J,1} & \cdot & \cdot & \cdot & x_{J,M} \end{bmatrix} \quad (3.4)$$

A least square fitting of the model equation is performed, in order to determine the values of coefficients c_1, c_2, \dots, c_M for which the model of equation 3.1 represents the series of J observations with the greatest accuracy. However, very often, in order to be in compliance with the requirements of accreditation and certification (EN ISO/IEC 17025), not only the coefficients, but also their variances and covariances are required for uncertainty analysis of results produced by any further use of the fitted model.

The deviations of the model from the real data can be attributed to experimental errors but also to model weaknesses. In any case, the basic working hypothesis considers the model to be suitable for the description of the related to experimental observations phenomena.

The basic methodology is almost always the same (Press et al., 1996): a *figure-of-merit function* is selected, to give an indication of the difference between the real data and the model. After this, the model parameters are selected so that the value of this function is minimized.

The most commonly used method for the fitting is this of the ordinary least squares (OLS), being also easy to implement. Ordinary least squares technique is based on a set of hypotheses that are not always fulfilled, mainly referring to the absence of errors on the values of the

independent variables and, often, to the homoscedasticity of the errors associated with the values of dependent variable. However, in practice, measured data are always subject to some uncertainty. Thus, through the procedure of best fit determination, as in the case treated here, a method is required for fitting the linear model to data with uncertainties in both dependent and independent coordinates.

In the case where only the values of the dependent variables are precisely known, the figure-of-merit function to be minimized presents the following form:

$$\chi^2 = (Y - Y_e)^T (u_Y^2)^{-1} (Y - Y_e) \quad (3.5)$$

Where u_Y^2 is the covariance (or uncertainty) matrix associated with the vector Y . Assuming that the covariance matrix u_Y^2 is diagonal, that is to say that the measurements Y_1, \dots, Y_J are independent one to the other, the relation 3.6 is written:

$$\chi^2 = \sum_{j=1}^J \left(\frac{y_j - (c_1 x_{j,1} + c_2 x_{j,2} + \dots + c_M x_{j,M})}{u_{k_j}} \right)^2 \quad (3.6)$$

For the case the uncertainties characterizing the values $x_{j,m}$ of the independent variables can not be considered as negligible, the figure-of-merit function to be minimized becomes (Lira, 2000):

$$\chi^2 = (X - X_e)^T (u_X^2)^{-1} (X - X_e) + (Y - Y_e)^T (u_Y^2)^{-1} (Y - Y_e) \quad (3.7)$$

where u_X^2 is the covariance (or uncertainty) matrix associated with the matrix X .

The derivation of equation 3.7 for the finding of its minimum value is quite complicated due to the non linear character it presents. For the needs of usual metrological applications, various approximate methods have been proposed, each one presenting different degree of difficulty. Within the context of the present investigation, the so-called “effective variance” approach is adopted, the implementation of which is justified in cases where the input uncertainty matrices can be considered as diagonal, as it is valid for the case concerning this work. According to this approach, the overall variance in each observation point results through the “transferring” of the uncertainties in X to those in Y and treating X as exactly known quantities (Lira, I.; Cecchi, 1991; Press *et al.*, 1996):

$$u_j^2 = \text{var}(y_j - (c_1 x_{j,1} + c_2 x_{j,2} + \dots + c_M x_{j,M})) = y_j^2 + c_1^2 u_{xj,1}^2 + c_M^2 u_{xj,1}^2 + \dots + c_M^2 u_{xj,M}^2 \quad (3.8)$$

In this case, the normal equation of the least square problem can be written:

$$(K^T \cdot K) \cdot C = K^T \cdot L \quad (3.9)$$

where C is a vector whose elements are the fitted coefficients, K is a matrix whose $J \times M$ components $k_{j,m}$ are constructed from M basic functions evaluated at the J experimental values of x_1, \dots, x_M weighted by the uncertainty u_j , and L is a vector of length J whose components l_j are constructed from values of y_j to be fitted, weighted by the uncertainty u_j :

$$k_{j,m} = \frac{x_{j,m}}{u_j}, \quad K = \begin{bmatrix} \frac{x_{1,1}}{u_1} & \cdot & \cdot & \cdot & \frac{x_{1,M}}{u_1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{x_{j,m}}{u_j} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{x_{J,1}}{u_J} & \cdot & \cdot & \cdot & \frac{x_{J,M}}{u_J} \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (3.10)$$

$$l_j = \frac{y_j}{u_j}, \quad L = \begin{bmatrix} \frac{y_1}{u_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{y_J}{u_J} \\ \cdot \end{bmatrix} \quad (3.11)$$

Given that for the calculation of variances u_j^2 the knowledge of coefficients c_1, c_2, \dots, c_M is needed, a possible solution is to use the values of coefficients calculated by ordinary least squares fitting as the initial values. These initial values can be used in equation 3.6 for the calculation of u_j^2 , $J=1 \dots J$ and the formation of matrix K and of vector L .

The solution of equation 3.12 gives the new values of coefficients c_1, c_2, \dots, c_M , which however are not expected to differ noticeably from those calculated by standard least squares fitting and used as initial values for the calculation of u_j^2 :

$$C = (K^T K)^{-1} (K^T L) \quad (3.12)$$

Moreover, $Z = \text{INV}(K^T \cdot K)$ is the uncertainty matrix whose diagonal elements $z_{k,k}$ are the squared uncertainties (variances) and the off-diagonal elements $z_{k,l} = z_{l,k}$, $k \neq l$ are the covariance between fitted coefficients:

$$u_{c_m} = \sqrt{z_{mm}}, \quad m=1, \dots, M \quad (3.13)$$

$$\text{Cov}(c_k, c_l) = z_{k,l} = z_{l,k}, \quad k=1, \dots, M \text{ and } l=1, \dots, M \text{ and } k \neq l \quad (3.14)$$

It should be noted that the knowledge of covariance between the fitted coefficients is necessary if one wishes to calculate, in a next stage, the uncertainty u_x in the predicted values of x using equations 3.1 and the law of error propagation (equation 2.3).

Equation 3.12 can be solved by a standard numerical method, for example, by Gauss-Jordan elimination. It is also possible to use matrix manipulation functions of commonly used spreadsheet software.

3.4. Uncertainty associated with expected solar collector efficiency

What most concerns future users of the collectors is the uncertainty characterizing the values of the collector efficiency, when this is calculated for given values of operation conditions (irradiance, angle of incidence, ambient temperature, inlet water temperature etc). The calculation of the expected efficiency y' can be easily done by entering in equation 1 the fitted coefficients $C = [c_1 \quad \dots \quad c_M]^T$ and the operation conditions $X' = [x'_1 \quad \dots \quad x'_M]$:

$$y = \sum_{m=1}^M c_m x'_m \quad \text{or in matrix notation } y' = X' \cdot C \quad (3.15)$$

The uncertainty in the predicted values of y can be calculated by applying the law of error propagation (equation 2.3), taking into account both variances and covariances of the fitted coefficient and considering the operation conditions X' to be known without uncertainties:

$$u_{y'} = \sqrt{\sum_{m=1}^M (x'_m u_{c_m})^2 + 2 \sum_{i=1}^{M-1} \sum_{j=i+1}^M x'_i x'_j \text{cov}(c_i, c_j)} \quad (3.16)$$

or in matrix notation:

$$u_{y'} = \sqrt{X' \cdot Z \cdot (X')^T} \quad (3.17)$$

3.5. Consistency of the fitting and uncertainty calculation

It emerges from equation 3.6 that the chi-square merit function actually gives an idea about the degree that the model deviation from the experimental data could be related with the measurement uncertainties. For a relatively good model, the deviations observed could be

attributed to the experimental errors and the corresponding χ^2 function will have a value close to the number of degrees of freedom.

More specifically, the minimum values of the figure-of-merit function χ^2 , denoted as χ_o^2 , follow approximately a chi-square distribution with $\nu=J-M$ degrees of freedom. Thus, it is reasonable to expect χ_o^2 to be close to $J-M$ (Lira, 2000). If $\chi_o^2 \ll \nu$, the deviations of the fitted model from the experimental data appear to be sensitively lower than the experimental uncertainties. Thus, it could be reasonable to conclude that the input uncertainty matrix is comparatively large, and it could be eventually feasible to investigate the potential of re-estimating the uncertainties downwards, without though such an action to be compulsory.

On the contrary, in the case where $\chi_o^2 \gg \nu$, the uncertainties characterizing the experimental inputs do not seem to be adequate for explaining the relatively larger deviations between the fitted model and the experimental measurements; possible explanations can be attributed to the presence of outliers in the measurements results, the inadequate number of observation points, or even the weakness of the used model to effectively describe the energy behaviour of the collector under test. If the improvement of the relation χ_o^2/ν can not be achieved, then the model (or the estimation procedure) should be called into question. Moreover, the uncertainty resulting from equation 3.16 should be enhanced, through the consideration of an additional component expressing the weakness of the model to describe the actual behavior of the collector.

An alternative way for the evaluation of the consistency of the fitting and of the uncertainty calculation which has been proposed in the relevant literature and leads to similar results is based on the calculation of the probability $Q(0.5\nu, 0.5\chi_o^2)$ that the data do not fit the model by chance (Press et al., 1996). This probability can be calculated by using the incomplete gamma function:

$$Q(a,x)=\frac{1}{\Gamma(a)}\int_x^\infty e^{-t}t^{a-1}dt \text{ with } a>0 \text{ and } \Gamma(a)=\int_0^\infty t^{a-1}e^{-t}dt \quad (3.18)$$

The probability Q can be explained as a quantitative indication of goodness-of-fit for the specific model. Generally speaking, if Q is larger than 0.1, then the goodness-of-fit is believable. If it is larger than 0.001, then the fit may be acceptable, under certain conditions. If Q is less than 0.001, then the model (or the uncertainty estimation procedure) can be called into question.

3.6. Example of method application

The results of the measurements through the testing of a solar collector according to EN12975-2 are presented in table 1 and figure1. As pointed previously, in the case of the steady state model, $M=3$, $y=n$, $c_1= \eta_0$, $c_2=a_1$, $c_3=a_2$, $x_1=1$, $x_2=(T_m-T_a)/G$ and $x_3=(T_m -T_a)^2/G$.

More specifically, in table 1 the following parameters are presented:

- The values of the efficiency indicator n (dependent variable) in cells C3:C36.

- The values of the independent variables $x_1=1$, $x_2=(T_m-T_a)/G$ and $x_3=(T_m -T_a)^2/G$ in cells D3:F38.
- The standard uncertainties characterizing the values of the dependent and independent variables, as these have been calculated by the primary measurements on the basis of the metrological characteristics of the measuring equipment used (cells G3:G38 and H3:J38 respectively). The uncertainty of the values of variable x_1 is not indicated, as it is zero (constant value $x_1=1$).

Table 3.1: Test data

a/a	y=n	$x_2= T^*$	$x_3= G(T^*)^2$	u_n	u_{T^*}	$u_{G \cdot T^{*2}}$
1	0,4671	0,0496	2,4771	0,0131	0,0013	0,0762
2	0,4687	0,0485	2,4198	0,0131	0,0013	0,0746
3	0,5709	0,0294	0,8847	0,0153	0,0009	0,0344
4	0,5647	0,0297	0,8913	0,0152	0,0009	0,0347
5	0,6516	0,0113	0,1291	0,0174	0,0005	0,0108
6	0,6480	0,0114	0,1313	0,0172	0,0005	0,0107
7	0,7035	-0,0003	0,0001	0,0185	0,0004	0,0004
8	0,7019	0,0000	0,0001	0,0184	0,0005	0,0032
9	0,7007	-0,0001	0,0001	0,0184	0,0004	0,0007
10	0,6993	-0,0001	0,0002	0,0184	0,0005	0,0019
11	0,6688	0,0079	0,0641	0,0176	0,0005	0,0074
12	0,6668	0,0078	0,0618	0,0176	0,0005	0,0071
13	0,5793	0,0276	0,7570	0,0155	0,0008	0,0317
14	0,5721	0,0289	0,8211	0,0154	0,0009	0,0331
15	0,4829	0,0478	2,3688	0,0132	0,0013	0,0730
16	0,4865	0,0478	2,3730	0,0133	0,0013	0,0733
17	0,4746	0,0510	2,6606	0,0130	0,0014	0,0810
18	0,4701	0,0512	2,6596	0,0129	0,0014	0,0807
19	0,5784	0,0304	0,9569	0,0154	0,0009	0,0362
20	0,5762	0,0305	0,9592	0,0153	0,0009	0,0363
21	0,6635	0,0116	0,1386	0,0175	0,0005	0,0110
22	0,6659	0,0112	0,1294	0,0175	0,0005	0,0107
23	0,7102	0,0008	0,0009	0,0185	0,0004	0,0009
24	0,7073	0,0012	0,0016	0,0185	0,0004	0,0012
25	0,7119	0,0009	0,0011	0,0186	0,0004	0,0011
26	0,7128	0,0009	0,0010	0,0186	0,0004	0,0010
27	0,6569	0,0108	0,1155	0,0174	0,0005	0,0100
28	0,6624	0,0104	0,1090	0,0174	0,0005	0,0097
29	0,5810	0,0289	0,8388	0,0155	0,0008	0,0334
30	0,5831	0,0284	0,8204	0,0156	0,0008	0,0328
31	0,4747	0,0486	2,3812	0,0130	0,0013	0,0737
32	0,4758	0,0481	2,3308	0,0130	0,0013	0,0726
33	0,4167	0,0588	3,5147	0,0118	0,0015	0,1028
34	0,4138	0,0581	3,4663	0,0118	0,0015	0,1011
35	0,4166	0,0579	3,4391	0,0117	0,0015	0,1007
36	0,4143	0,0593	3,5489	0,0117	0,0015	0,1034

- The initial values of the coefficients, to be used in equation 11 (cells I42:K42). This value is calculated through the ordinary least square method (**LINEST** function of excel).

- The total standard deviation u for every observation point, according to relation 3.3 (cells K3:K38).
- The values of matrix K and vector L , according to relations 3.10 and 3.11 (cells C43:C78 for vector L and D43:F78 for matrix K).
- The final values of the coefficients, as calculated by equation 3.12 through the use of the excel function (cells J46:J48, excel formulae: ***MULT(MINVERSE(MMULT(TRANSDPOSE(D43:F78);D43:F78));MMULT(TRANSDPOSE(D43:F78);C43:C78))***).
- The coefficient's uncertainty matrix Z (cells I53:K55, excel formulae: ***MINVERSE(MMULT(TRANSDPOSE(D43:F78);D43:F78))***).
- The coefficient's uncertainties (cells J58:J60), as the square root of the diagonal elements of the uncertainty matrix Z , and their covariance's (cells L58:L60), as the off-diagonal elements of the same matrix.
- The consistency check (cells J65:K66), based on the comparison of the minimum value of figure-of-merit function x^2 with the number ν of degree of freedom.
- A calculation example, for $G=800 \text{ Wm}^{-2}$ and $T_m-T_a=30 \text{ K}$, of the associated standard uncertainty u_n (cell J74) and the associated expanded uncertainty U_n of the expected efficiency (cell J73), for a coverage probability of 95% (cell J76).

4. Uncertainties in solar thermal systems efficiency testing

4.1. General

The expected energy output presents the most important quantity, within those quantities used for the energy characterization of the systems utilizing renewable energy sources in general, and solar domestic hot water systems (SDHWS) more specifically. The continuously increasing penetration of energy certification schemes and the connection of the expected energy gains with supporting actions, motivated by the demand on the users part for reliable performance data of solar thermal products, makes necessary the estimation of uncertainty characterizing the test results. This estimation should be performed on the basis of a commonly accepted, scientifically sound methodology.

The quantitative estimation of the energy behaviour of a SDHWS is performed according to the valid international standards, through the implementation of specific experimental sequences, involving measurement of climatic data and energy related system quantities (CEN, 2006; ISO, 1995). The exploitation of the measurements data for the calculation of the expected annual energy output involves calculations on multiple stages, noting that it is not possible to formulate explicitly a measurement model for the connection of primary experimental testing data with the calculated result. The absence of such a model leads to specific difficulties as regards the estimation of uncertainty on the final energy result, to the degree that it makes the adoption of the conventional approach for error propagation impossible.

Standard EN12976-2 proposes two methods for the performance testing of SDHWS, the DST and CSTG method, on the basis of two different approaches for the modeling of the energy behaviour of the systems (CEN, 2006). The two methods are considered equivalent, while a coefficient for the conversion of the results of the one method to the other is proposed.

The method CSTG is based on a relatively simple model of energy behavior, which treats the SDHWS as a black-box, while the model of the DST method is more detailed and involves the characteristics of the individual components.

The evaluation of the performance of the one or the other method exceeds the scope of the present study. It should be noted though that, as regards the estimation of uncertainties, the DST method presents some additional difficulties related to the energy model used, as this model is not exactly known and the calculations implemented by the respective software are not known to the user. On the contrary, the steps which have to be implemented during the application of the CSTG method are transparent and clearly defined, while the relevant software is clearly described in the Standard. On this point of view, the analysis for the estimation of uncertainties is different for each one of the two methods.

4.2. The CSTG testing method

4.2.1. Non-algebraic model and Monte-Carlo simulation

The quantitative estimation of the energy behaviour of a SDHWS is performed according to

the valid international standards, through the implementation of specific experimental sequences, involving measurement of climatic data and energy related system quantities (CEN, 2006; ISO, 1995). The exploitation of the measurements data for the calculation of the expected annual energy output involves calculations on multiple stages, noting that it is not possible to formulate explicitly a measurement model for the connection of primary experimental testing data with the calculated result. The absence of such a model leads to specific difficulties as regards the estimation of uncertainty on the final energy result, to the degree that it makes the adoption of the conventional approach for error propagation impossible (see section 2).

The objective of this work is the estimation of uncertainty characterizing the expected annual energy output of a SDHWS, as calculated through the tests performed according to the valid international standard (ISO, 1995). The adoption of the respective approach is justified by the fact that the standard proposes validated and internationally accepted testing and calculation procedures, which are also recognized by market actors and potential users.

For the estimation of the effect of the metrological quality of used measurement equipment, Monte-Carlo simulation techniques are exploited. In the analysis the uncertainty related to the imperfections of the energy model for the behaviour of SDHWS assumed by the standards is counted in, as well as the uncertainty attributed to the natural variance of the meteorological data. The final scope is the provision, to the future user of the test results of a SDHWS, of sufficient and reliable information regarding the energy benefit by the operation of the system in actual working conditions.

It is noted that, even if the potential user's confidence in performance indicators of renewable energy systems is of high importance, no specific investigation for SDHWS has been recorded in the relevant literature, opposite to the case of solar collectors (Li and Lu, 2005; Mathioulakis et al., 1999). A previous work is mentioned, proposing a systematic investigation of the parameters affecting the quality of the test result and aiming at the optimization of the test method itself, without, though, proposing assessment of uncertainties in compliance with up-to-date metrological concepts for the estimation of metrological uncertainty (Bourges et al., 1991).

For the needs of the present analysis, the case of a performance test of a typical SDHWS, with a collector surface of 3.76 m² and tank volume of 191 l, has been selected. Measurements have been performed with calibrated equipment, and the anticipated procedures by the EN12976-2 Standard have been strictly followed. Even though the specific quantitative results concern the tested solar system, the proposed methodology can be implemented for any other type of SDHWS. Moreover, any deviation from the requirements of the standard (e.g. different energy model or load profiles) does not exclude the implementation of the proposed methodology, even though the final results may be influenced.

In 4.2.2 the testing method is presented with emphasis on the propagation of information from the primary experimental testing data to the final result of the expected energy output. 4.2.3 deals with the methodology proposed for the estimation of uncertainty by examining the different components in detail. In 4.2.4, results concerning a typical SDHWS are presented, while in 4.2.5 basic conclusions are discussed.

4.2.2. Test method and measurement model

According to the CSTG method, thermal energy Q , accumulated on the storage tank of a SDHWS by the duration of the day, is correlated to the incident daily solar radiation on the

collectors surface H , the mean daily ambient temperature \bar{T}_a and the temperature T_{sin} of the tank at the beginning of the day, through the characteristic equation of the system (Belessiotis et al., 2010; CEN, 2006; ISO, 1995):

$$Q = a_1 H + a_2 (\bar{T}_a - T_{sin}) + a_3 \quad (4.1)$$

The estimation of the expected energy output is performed on two stages (Figure 4.1). At the first stage, that of testing, specific experimental scenarios are realized, aiming at the determination of specific energy characteristics of the solar system, as the coefficients a_1 , a_2 and a_3 of the characteristic equation, the heat losses coefficient of the storage tank U_s , as well as the two dimensionless draw-off profiles, h and g , characterizing the distribution of temperature on a homogenized and non-homogenized tank respectively.

The coefficients a_1 , a_2 and a_3 of the characteristic equation are determined by the multi-factor least-squares method, on the basis of a series of daily tests. According to the scenarios of these tests, the system begins operation in the morning, the storage tank being on a known initial temperature, which would be the temperature of the tank at the end of the day, after the drawing off of the thermal energy Q accumulated during the daytime.

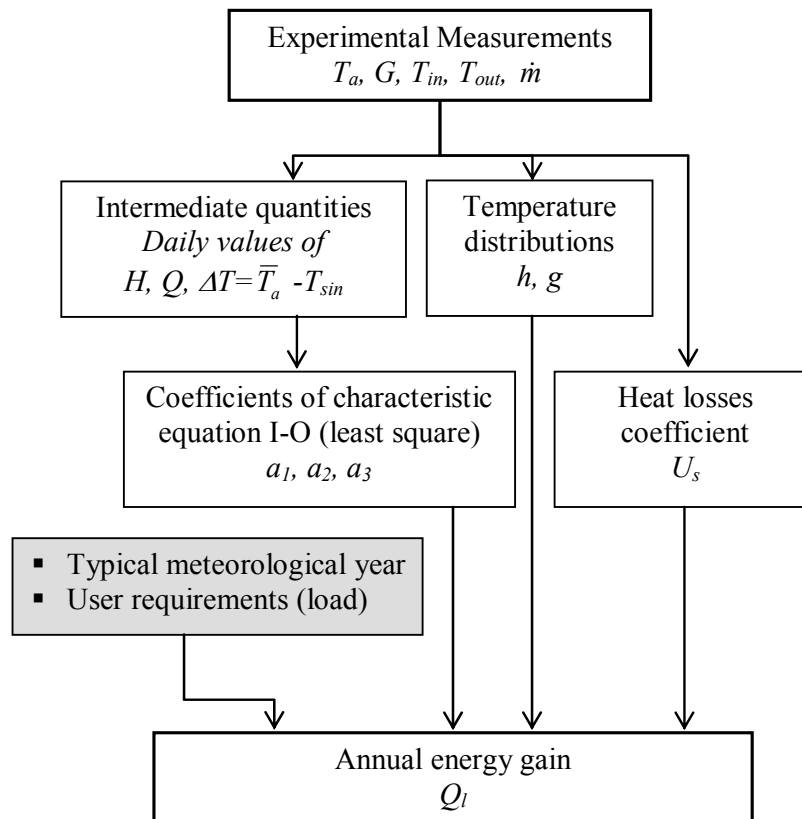


Figure 4.1. Flow-chart of actions required for the calculation of the expected energy output

Energy H is calculated by arithmetic integration of the instantaneous solar radiation G through

the duration of the day. The daily useful energy gain Q is calculated by integration, considering the draw-off flow-rate \dot{m} and the temperatures T_{in} and T_{out} of the water on the inlet and outlet of the tank respectively:

$$Q = \int \dot{m} C_p (T_{out} - T_{in}) dt \quad (4.2)$$

For the calculation of the heat losses coefficient, U_s , the tank is initially heated up to a homogeneous temperature T_i , and stays still for a time period δt of about 12 hours. By the end of this period, the tank is homogenized and the final temperature T_f is measured. Knowledge of the thermal capacity of the storage, mC_p , and the mean ambient temperature \bar{T}_a during the test, enables calculation of the heat losses coefficient through the following relation:

$$U_s = \frac{mC_p}{\delta t} \ln \frac{T_i - \bar{T}_a}{T_f - \bar{T}_a} \quad (4.3)$$

It is noted that temperatures T_i and T_f are measured by sensors placed on the inlet and outlet of the storage tank, which is also the case for the determination of the non-dimensioned distributions h and g .

At the second stage, that of the expected energy output calculation, the energy characteristics identified through the tests are used for the calculation of the expected energy output. Calculations are performed for a specific site, where the system is expected to be installed, for conditions determined by the Typical Meteorological Year (TMY) of this area and for specific hot water use patterns. The calculation is based on a procedure explicitly determined by the Standard, according to the following steps:

- I. For each one of the 365 days of the year, the expected accumulated energy on the solar tank during the day is calculated by equation 4.1, considering the storage temperature by the beginning of the day and the TMY data.
- II. The remaining energy in the tank by the end of the day is calculated after subtracting the thermal energy consumed by the user.
- III. The available energy by the beginning of the following day is determined after counting the heat losses by the night time.
- IV. Return to step 1, and continuation of calculations for the following day, until the end of the year.

The whole procedure involves a series of discrete calculation activities, the combination of which can be considered as the measurement model (Figure 4.2). The primary data that the measurement model uses as inputs, are the ambient temperature T_a , temperatures on the inlet and outlet of the storage tank T_{in} and T_{out} respectively, the flow rate of water on the outlet of solar tank, as well as the instantaneous solar radiation flux G . The output of the measurement model is the expected annual energy production, Q_l .

It is noted that the measurement model describes the relation of the final result to the primary experimental data, and should not be confused with the energy model which describes the energy behaviour of the system, as formulated by equation 4.1.

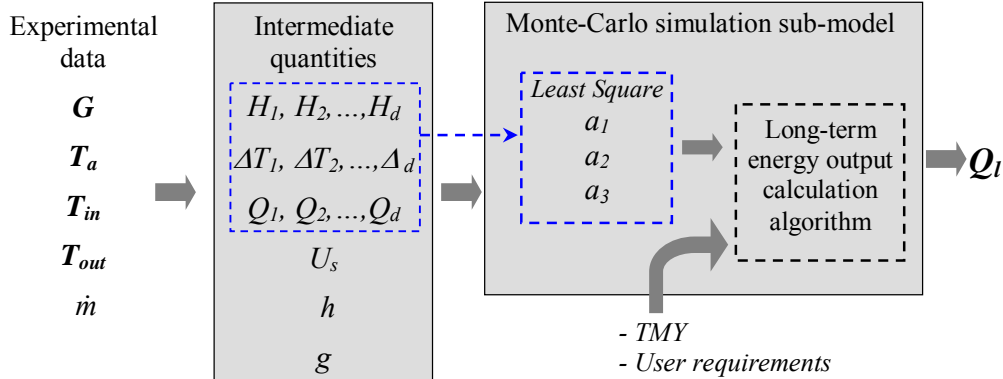


Figure 4.2. Correlation of the result to primary experimental test data (measurement model)

In terms of the estimation of uncertainties, the error propagation approach can be implemented for the determination of uncertainties related with the intermediate quantities. Nevertheless, the part of the measurement model connecting the energy output Q_l with the intermediate quantities, cannot be formulated as a set of derivable equations, thus the error propagation approach cannot be implemented. In order to overcome this limitation, the Monte-Carlo technique can be adopted for this part of the model (see section 2 and Figure 2.1).

4.2.3. Sources of uncertainty and influence on the final result

4.2.3.1. Assumptions and methodological approach

It is commonly accepted that the uncertainty associated with the result of a measurement, is not a property of the quantity under measurement but it characterizes the measurement method and the metrological quality of the measuring equipment (BIPM et al., 2008a). For reasons related to the capability of generalizing the conclusions of the specific investigation, the assumption that the used measuring equipment complies with the metrological requirements of the Standard is made. In case a laboratory achieves higher metrological performance, the uncertainties can be reassessed through the same methodological approach. It is also noted that:

- The assumption that the requirements of the Standard regarding the implementation of the test method are satisfied is made. Potential deviations from the method can introduce additional uncertainty components, which should be assessed case dependently. These deviations could concern the testing scenarios, the energy model of the system (equation 4.1), or the adoption of different load profiles.
- The calculated uncertainties concern the expected energy output of a SDHWS identical to the tested one. Even though other systems of the same type would present slightly different performance, experience shows that in the case of products coming out of the same production line, these differences are quite limited and do not constitute a remarkable source of uncertainty.

- The load pattern implemented by the user of a SDHWS predominantly influences the expected energy output, and, in consequence, the uncertainty characterizing this figure. Within the context of the present work, the energy accumulated in the solar tank is considered to be drawn-off by the end of each day of the year, as anticipated in the standard (CEN, 2006). A large volume of water drawn-off is considered (3 times the volume of the tank), a choice which is of higher interest to the user, to the degree that it practically corresponds to the maximization of energy gain through the use of the system. However, the influence on the expected energy output of assuming different values for the volume of the water drawn-off is investigated, as it will be shown later on.

According to the proposed approach, the calculation of the uncertainty characterizing the expected annual energy output is implemented on five distinct steps, each of them corresponding to specific activities anticipated by the standardized testing method:

I. Initially, the uncertainties related to the intermediate quantities H , ΔT , Q and U_s are estimated, on the basis of the metrological characteristics of the measuring equipment, as explained in section 4.2.3.2. On the basis of the provided information, a stochastic image is built for each one of the values of H , ΔT , Q and U_s , as described in sections 2.2 and 4.2.3.2.

II. The stochastic information which has been produced for the experimental values of intermediate quantities is propagated through the measurement model, as described in section 4.2.3.3, aiming at the estimation of both the expectation and the variance of the calculated energy output.

III. The uncertainty component related to the imperfections of the energy model (equation 4.3), i.e. its inability to explain precisely the experimental data gathered during the test, is estimated as discussed in section 4.2.3.4.

IV. The uncertainty related to the fact that the meteorological conditions which may occur during the actual operation of the SDHWS system cannot be precisely known, is estimated as described in section 4.2.3.5.

V. Finally, the combined uncertainty characterizing the final result is computed as the square root of the algebraic sum of the individual variance contributions (BIPM et al., 2008a).

All calculations have been performed in a MATLAB environment. Given that the procedure used in the paper is complex and a large number of data is involved, special treatment regarding the checking of the validity of calculations has been given. This included mainly the comparative checking of results through the proper modification of different sources of uncertainty, as well as the comparison of the mean values of the expected energy output to the results provided by the software described in the standard (ISO, 1995).

4.2.3.2. Calculation of uncertainties of intermediate quantities

4.2.3.2.1. Uncertainties characterizing the mean temperature \bar{T}

In the case the temperature values T_1, T_2, \dots, T_n are available, the mean temperature \bar{T} is calculated by the following relation, which represents the measurement model as well:

$$\bar{T} = \frac{T_1 + T_2 + \dots + T_n}{n} \quad (4.4)$$

For the calculation of the uncertainty in the value of the mean temperature \bar{T} , the existence of potential correlations between the values collected during the day has to be taken into consideration. In the specific case examined in this work, the usual metrological practice for relevant cases has been adopted, i.e. the maximum value for the uncertainty is selected (a worst case scenario), the errors on the collected values being considered as fully correlated to one another, as the individual values have been taken by the same instrument (BIPM et al., 2008a). Assuming that the uncertainty is the same for all measurement points, equal to u_T , the implementation of the error propagation law leads to:

$$u_{\bar{T}} = \sqrt{\sum_{i=1}^n \left(\frac{\partial T}{\partial T_i} u_{T_i} \right)^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{\partial T}{\partial T_i} \frac{\partial T}{\partial T_j} u_{T_i} u_{T_j}} = \sqrt{\frac{1}{n^2} \left(\sum_{i=1}^n u_T \right)^2} = u_T \quad (4.5)$$

4.2.3.2.2. Uncertainty characterizing the daily integral of solar radiation H

Let the solar radiation measurements G_1, G_2, \dots, G_{n+1} be sampled with a time step δt , for a total duration of $\Delta t = n \times \delta t$. Assuming that the sampling speed is suitable for the accurate representation of the solar radiation variation, the daily solar energy H can be calculated according to the following relation:

$$H = \delta t \sum_{i=1}^n G_i \quad (4.6)$$

The degree of correlation between the values collected during the daytime has to be taken into consideration. Similarly to the case of temperature, the values G_1, G_2, \dots, G_{n+1} , can be considered as correlated to one another (a worst case scenario). Given that the time step δt is known with negligible uncertainty, the implementation of the error propagation law leads to:

$$u_H = \sqrt{\sum_{i=1}^n \left(\frac{\partial H}{\partial G_i} u_{G_i} \right)^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{\partial H}{\partial G_i} \frac{\partial H}{\partial G_j} u_{G_i} u_{G_j}} = \sqrt{(\delta t)^2 \left(\sum_{i=1}^n u_{G_i} \right)^2} = \Delta t u_G \quad (4.7)$$

4.2.3.2.3. Uncertainty characterizing the daily energy output Q

Let Q be the thermal energy subtracted from the tank by the end of a testing day, calculated by integration of the n measured values of the temperature difference in the inlet and outlet of the solar thermal system and the fluid flowrate \dot{m} , over the whole draw-off period $\Delta t = \sum \delta t$:

$$Q = \int \rho C_p \dot{m} (T_{out} - T_{in}) dt = \rho C_p \delta t \sum_{i=1}^n \dot{m} (T_{out} - T_{in}) \quad (4.8)$$

Given that the uncertainties of the density, specific heat capacity values and time step δt can

be considered negligible, and by making the assumption of totally correlated errors, the following relation for the uncertainty of energy output can be formulated:

$$u_Q = \sqrt{\left(\frac{\partial Q}{\partial(T_{out} - T_{in})} u_{T_{out} - T_{in}}\right)^2 + \left(\frac{\partial Q}{\partial \dot{m}} u_{\dot{m}}\right)^2} = \rho C_p \delta t \sqrt{\dot{m} u_{T_{out} - T_{in}} + (T_{out} - T_{in}) u_{\dot{m}}} \quad (4.9)$$

4.2.3.2.4. Uncertainty characterizing the heat loss coefficient U_s

The storage tank heat loss coefficient is calculated according to relation (4.5), through the implementation of the error propagation law, considering the uncertainties of specific heat capacity and time values as negligible:

$$u_{U_s} = \frac{m C_p}{\Delta t_u} \sqrt{\left(\frac{u_{T_i}}{T_i - \bar{T}_a}\right)^2 + \left(\frac{u_{T_f}}{T_f - \bar{T}_a}\right)^2 + \left(\frac{u_{\bar{T}_a}}{T_f - \bar{T}_a} - \frac{u_{\bar{T}_a}}{T_i - \bar{T}_a}\right)^2} \quad (4.10)$$

Finally, an extended sensitivity analysis has shown that the contribution of the uncertainty related to the non-dimensionless profiles h and g can be considered negligible, compared to the other sources of uncertainty, so it can be neglected.

4.2.3.3. Uncertainty component related to the errors of the sensors

As it has been previously indicated, the rather complicated non-algebraic calculation of the expected energy output makes the Monte-Carlo simulation a suitable choice. This approach is implemented in practice through the following steps:

- I. The implementation of the experimental scenarios anticipated by the testing procedure of the system leads to a matrix \bar{D} containing the d daily values H_i , ΔT_i and Q_i , $i=1, \dots, d$, for the quantities H , ΔT and Q :

$$\bar{D} = \begin{bmatrix} H_1 & \Delta T_1 & Q_1 \\ H_2 & \Delta T_2 & Q_2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ H_d & \Delta T_d & Q_d \end{bmatrix} \quad (4.11)$$

- II. On the basis of the metrological characteristics of the sensors used, the uncertainties characterizing U_s , as well as the uncertainties of all the elements of the matrix D are calculated as described in paragraph 4.2.3.2.
- III. For each one of the elements of the matrix \bar{D} , as well as for U_s , a large number of

random values is produced, the statistical properties of which are identical with the metrological characteristics of the simulated quantity.. More specifically, the expectation and the standard deviation of the probability distribution assigned to each quantity, are equal to the measured value of this quantity and the associated standard uncertainty respectively. As a random number generator, the Mersenne Twiste algorithm has been used (Matsumoto and Nishimura, 1998), and the assumption that all quantities follow the normal distribution is made. A value of $N=10^6$ has been selected, since this number of trials is expected to deliver a value of expanded uncertainty which is correct to one or two significant decimal digits (BIPM, 2008b).

- IV. As resulting from step III, N distinct configurations $\bar{D}_j, j=1, 2, \dots, N$ of the matrix \bar{D} are produced. For each configuration \bar{D}_j , combined with a random value $U_{s,j}$ of the coefficient U_s , and the dimensionless draw-off profiles, h and g , a value $Q_{l,j}$ of the expected energy output Q_l is produced:

$$\bar{Q}_l = \begin{bmatrix} Q_{l,1} \\ Q_{l,2} \\ \cdot \\ \cdot \\ \cdot \\ Q_{l,N} \end{bmatrix} = \begin{bmatrix} f(\bar{D}_1, U_{s,1}, h, g) \\ f(\bar{D}_2, U_{s,2}, h, g) \\ \cdot \\ \cdot \\ \cdot \\ f(\bar{D}_N, U_{s,N}, h, g) \end{bmatrix} \quad (4.12)$$

- V. From the $N=10^6$ values of the expected energy output produced, the mean value Q_l and the standard deviation $u_{Q_l, meas}$ are calculated, the latter considered as an efficient estimation of the standard uncertainty associated with the mean value.

4.2.3.4. Imperfections of the energy model and related uncertainty

The implemented CSTG model correlates the daily energy output Q to the daily incident solar energy H and the temperature difference $\Delta T = T_a - T_{sin}$. The specific model, as any model of this type, can be considered to some degree as approximate. Even though equation 4.1 describes satisfactorily the energy behaviour of the system, it is evident that some kind of error is introduced through its use, potentially affecting the quality of the results of the method.

The component of uncertainty related to the imperfections of the energy model, expresses the degree the model can explain the experimental data. Thus, the effectiveness of the CSTG model can be quantified by the standard estimation error of the linear fitting:

$$\sigma_Q = \left[\frac{1}{d-3} \sum_{i=1}^{i=d} (Q_i - (a_1 H_i + a_2 \Delta T_i + a_3))^2 \right]^{0.5} \quad (4.13)$$

where Q_i, H_i and ΔT_i are the values measured during the d testing days of the system, and a_1, a_2 and a_3 the values of the coefficients determined through the least squares approach.

Nevertheless, the typical prediction error can be entirely attributed to the imperfections of the model, only in the case that the experimental observations are free of any errors. By the presence of experimental errors, the standard error can partially represent the imperfections of the model, as well as the ones of the measurement equipment. On this point of view, for cases where the linear fitting is repeated into every one of the Monte-Carlo simulations with different standard prediction error, as the one examined in this work, the study of the variance of the prediction error can lead to useful conclusions.

If this standard error would remain practically constant from simulation to simulation, it could be promptly concluded that no significant influence of the measurement errors takes place. In the opposite case, that of the standard error varying as the coefficients change stochastically, it can be stated that this error depends on the measurement errors to some degree. Since it is difficult to separate these two sources of error (model and sensor imperfections), consideration of two independent components of uncertainty is necessary (a worst case scenario). In the case of a repeated Monte-Carlo simulation, the specific component of uncertainty $u_{Q_l, \text{mod}}$ can be reasonably considered equal to the average value of the standard errors recorded through the N simulations.

4.2.3.5. Uncertainty related to the variance of the meteorological conditions

As it has been previously noted, the expected energy output is calculated for a Typical Meteorological Year (TMY), statistically representative of the climatic conditions expected to take place on the installation site of the SDHWS (BIPM et al., 2006; Gazela and Mathioulakis, 2001). Nevertheless, the actual energy gain for the potential user strongly depends on the meteorological conditions which may occur during the actual operation of the system. Since these conditions are a priori different from the ones included in the TMY, an additional source of uncertainty related to the energy output value has to be considered.

In principle, the variability of meteorological data could be introduced in the Monte-Carlo simulations in the form of a statistical distribution; nevertheless such a statistic is hard to be determined. Moreover, the computational requirements would be difficult to satisfy, as the resulting number of combinations for the Monte-Carlo simulations would be very high.

In the present work, the actual meteorological data of a significant number of years for the geographical area of Athens have been used for the estimation of the variance of the expected energy output due to the variation of meteorological conditions. For each of these meteorological years, the expected energy output of the system is calculated. Assuming that Y different years are used, the respective component of the relative uncertainty can be estimated by the standard deviation of the total $i=1, \dots, Y$ energy output results:

$$u_{r, Q_l, \text{meteo}} = \sqrt{\frac{1}{Y-1} \sum_{i=1}^Y (Q_{l,i} - \bar{Q}_l)^2} / \bar{Q}_l \quad (4.14)$$

4.2.4. Arithmetical application

4.2.4.1. SDHWS and experimental measurements

The presented methodological approach, has been implemented for the case of a typical

domestic hot water system, with a collector aperture surface $A_c=1.86 \text{ m}^2$ and solar tank volume $V_s=177 \text{ l}$. The system has been tested through the strict implementation of the standard requirements regarding the testing procedure, as well as the metrological quality of the calibrated sensors used.

The metrological quality of the measurement setups which have been used, being compatible with the requirements of the standard, can be formulated on terms of standard uncertainty as follows:

- For the temperature of the fluid and the difference of temperature between the inlet and outlet of the system: $u_T = u_{T_{out}-T_{in}} = 0.06 \text{ K}$.
- For the ambient temperature: $u_{T_a} = 0.29 \text{ K}$.
- For the flow rate of the thermal medium: $u_{\dot{m}} = 0.58 \%$.
- For the incident solar radiation (by using a first class pyranometer): $u_G = 2.5 \%$

It is noted that within valid international standards the metrological requirements are formulated on accuracy terms. In these cases, the calculation of standard uncertainty u_A of a quantity A , is based on the consideration of an orthogonal probability distribution, correlating this uncertainty to the respective accuracy a_A through the relation:

$$u_A = \frac{a_A}{\sqrt{3}} \quad (4.15)$$

Table 4.1: Values of intermediate quantities and respective standard uncertainty

Day Number	Q [MJ]	u_Q [MJ]	ΔT [K]	$u_{\Delta T}$ [K]	H [MJ/m ²]	u_H [MJ/m ²]
1	37.2	0.29	-7.5	0.29	23.2	0.58
2	35.7	0.28	-6.6	0.29	22.0	0.55
3	30.0	0.25	-6.7	0.29	18.5	0.46
4	34.9	0.28	-3.5	0.29	20.5	0.51
5	30.1	0.26	2.2	0.29	16.6	0.42
6	24.2	0.23	0.0	0.29	13.3	0.33
7	23.0	0.22	-1.6	0.29	12.9	0.32
8	33.1	0.27	0.1	0.29	18.1	0.45
9	22.7	0.22	-1.5	0.29	12.7	0.32
10	21.9	0.21	-2.7	0.29	12.1	0.30
11	26.8	0.24	-4.3	0.29	16.1	0.40
12	28.6	0.25	-3.2	0.29	16.6	0.42
13	32.7	0.27	-3.5	0.29	18.9	0.47
14	25.9	0.23	-4.2	0.29	15.0	0.38
15	21.8	0.21	-5.7	0.29	12.7	0.32
16	16.9	0.19	-2.4	0.29	9.8	0.24
17	27.6	0.24	0.1	0.29	15.5	0.39
18	18.7	0.20	2.0	0.29	10.1	0.25
19	16.1	0.19	0.0	0.29	8.7	0.22
20	25.0	0.23	-0.3	0.29	14.0	0.35
21	25.0	0.23	-2.0	0.29	13.9	0.35
22	26.7	0.24	-6.6	0.29	16.8	0.42
23	23.0	0.22	-8.6	0.29	15.0	0.38
24	30.8	0.26	-2.9	0.29	18.0	0.45
25	30.1	0.26	-1.6	0.29	17.4	0.44

Especially for the measurement of solar radiation, instead of proposing an accuracy limit for the measuring instrument, the testing Standard sets the requirement of using pyranometers belonging on the first Class category or better. According to the recommendations of the World Meteorology Organization (WMO), the expanded uncertainty achieved by a first Class pyranometer for the measurement of daily radiation is 5% at a confidence level of 95%, thus the respective standard uncertainty lies in the order of 2.5% (WMO, 2008).

From the total number of the experimental measurements, 25 daily values for H , ΔT and Q have been selected with a view to the balance between days of low and days of high radiation. According to what has been mentioned in paragraph 4.2.3.2, the values of the intermediate quantities have been calculated, as well as the standard uncertainties characterizing these values (Table 4.1). The coefficient of storage tank heat losses U_s has been calculated as equal to 2.10 W/K, with a standard uncertainty of 0.04 W/K.

The predicted energy output Q_I has been calculated for different load profiles of use, their difference being the quantity of hot water drawn off on a daily basis. According to the Standard, the calculation of the predicted annual energy output Q_I is implemented for a daily draw-off of hot water on a temperature equal to 45°C by the end of the day. Thus, for each calculation of the annual energy output, the amount of water drawn-off every day as well as the required temperature, remain constant during the whole year. Calculation of annual energy output has been repeated for different volumes of daily water consumption, ranging from 1 to 3 times the volume of the tank, the closer to the upper value of 3 times the closer to the complete draw-off of the collected by the system solar energy.

4.2.4.2. Uncertainty component related to the errors of the measuring set-ups

The Monte-Carlo approach has been implemented for the estimation of the value of Q_I and the related uncertainty, according to the procedure described in 4.2.3.3. A specific hot water use profile has been adopted, and the geographical area of Athens has been considered as the installation site. A number of $N=10^6$ simulations have been performed, on the basis of data presented in table 4.1.

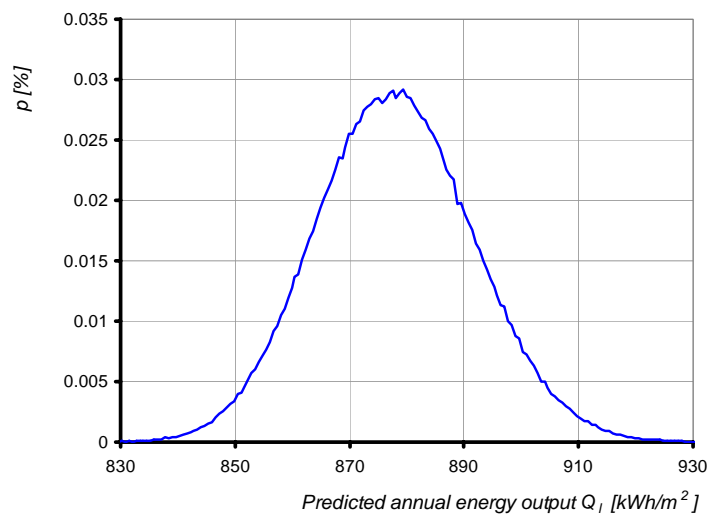


Figure 4.3: Probability distribution of Q_I for draw off volume equal three times the volume of the tank

It has been considered that the existing information provided by each experimental observation is expressed satisfactorily by a normal probability distribution, the expectation of which being the measured value and the variance being the squared standard uncertainty associated with this value. Simulations lead to 10^6 values for the expected energy output Q_l . The statistical elaboration of these values leads to the most likely value, which is estimated by the mean value, as well as of the standard uncertainty, which is estimated by the standard deviation.

In Figure 4.3, a characteristic probability distribution for draw off volume equal three times the volume of the storage tank is presented, while in Table 4.2 the values of the expected energy output for different profiles of hot water use are presented, the energy output being calculated as the mean value of the respective probability distributions. In the same table, the respective uncertainty values are presented, expressed as standard uncertainty $u_{Q_l,meas}$ and

relative standard uncertainty $u_{r,Q_l,meas} = \frac{u_{Q_l,meas}}{Q_l}$.

Table 4.2: Expected energy output Q_l , standard uncertainty $u_{Q_l,meas}$ and relative uncertainty $u_{r,Q_l,meas}$ for different values of draw-off volume V_d

Draw-off volume V_d	$3 V_s$	$2.5 V_s$	$2 V_s$	$1.5 V_s$	V_s
Q_l [kWh/m ²]	875	850	776	657	474
$u_{Q_l,meas}$ [kWh/m ²]	14	13	11	9	6
$u_{r,Q_l,meas}$ [%]	1.6	1.5	1.4	1.4	1.4

4.2.4.3. Uncertainty component related to the imperfections of the energy model

The uncertainty component related to the weakness of the energy model to sufficiently explain the experimental data has been estimated with regard to the approach described in 4.2.3.4. In Figure 4.4, the probability distribution of the standard error σ_Q of the energy model of the system is depicted, as calculated by the $N=10^6$ Monte-Carlo simulations.

The variation of the standard error from one simulation to the other indicates that the inability of the model (equation 4.1) to precisely explain the experimental data can be attributed, to some degree, to potential errors of the measuring equipment. Nevertheless, as it has been previously mentioned, the difficulty of separating these errors from the imperfections of the model itself, inevitably leads to the consideration of standard error as a discrete source of uncertainty (a worst case scenario).

The uncertainty component $u_{Q_l,mod}$, related to the suitability of the model, has been calculated as of 0.24 kWh/day, for draw-off volume three times the one of the tank, and consequently on an annual basis $u_{Q_l,mod}=23$ kWh/m². Expressed on relative uncertainty terms, the value of this component is $u_{r,Q_l,mod}=2.6\%$ of the expected energy output. It is noted that the value of this uncertainty component remains practically constant, regardless of the hot water drawn-off volume.

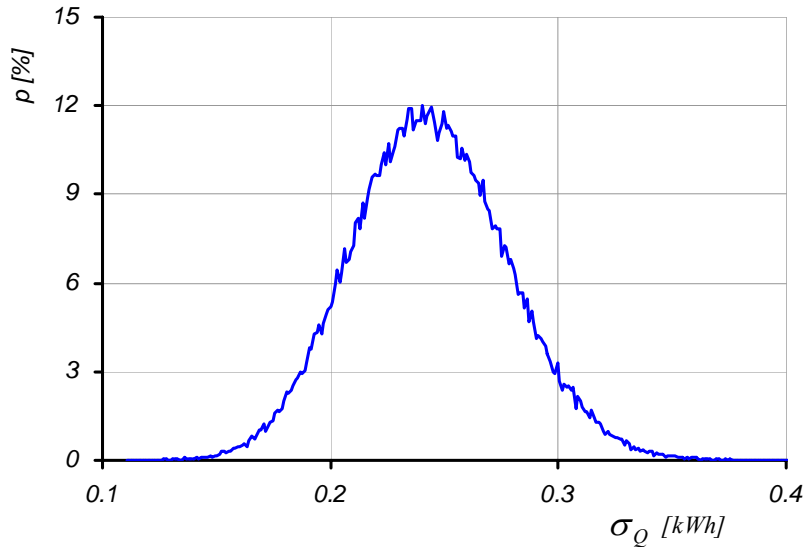


Figure 4.4: Probability distribution of standard error

4.2.4.4. Uncertainty component related to the variation of the meteorological conditions

The uncertainty component related to the variation of the meteorological conditions has been estimated according to the approach presented in 4.2.3.5. The annual energy output for 20 consecutive years has been calculated, by using the meteorological data of the period from year 1989 to year 2008, for the geographical area of Athens (courtesy of the Institute of Environmental Research and Sustainable Development - National Observatory of Athens). The calculation has been implemented on the basis of the calculation method anticipated in Standard EN 12976-2 (CSTG method).

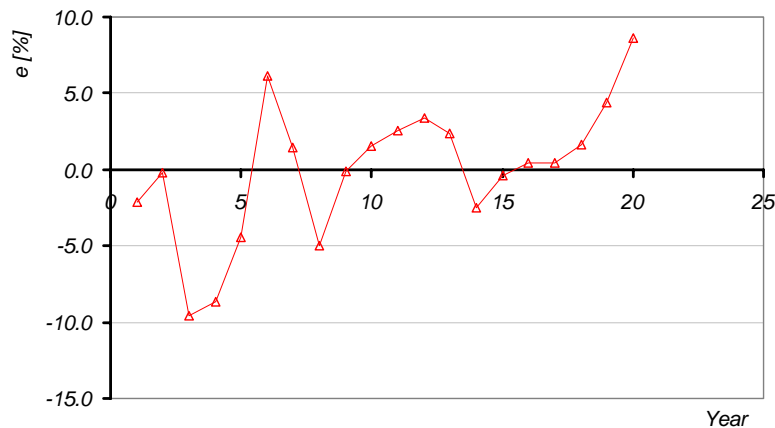


Figure 4.5: Deviations of the annual energy output for 20 different meteorological years from the 20-year mean value

Typical results for a draw-off volume $V_d=3V_c$ are presented in Figure 4.5 in terms of deviation from the 20-years mean values, while the related uncertainty component, calculated according to equation 4.14, is presented in Figure 4.6 for different draw-off volumes. The component of

relative uncertainty is estimated to be in the order of $u_{r,Q_L,meteo}=3.5\%$, a value remaining practically unchanged by the variation of the hot water volume drawn-off.

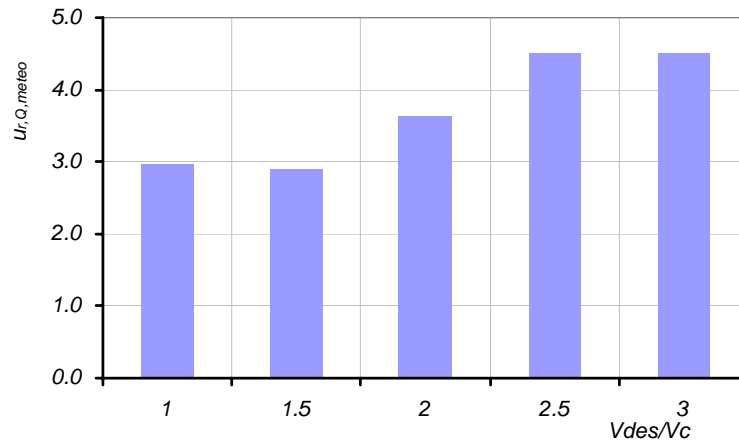


Figure 4.6: Relative uncertainty of the annual energy output for 20 different meteorological years with regard the hot water volume drawn-off

4.2.4.5. Total combined uncertainty

The combined standard uncertainty associated with the values of the annual expected energy output is calculated through the combination of the individual components:

$$u_{r,Q_L} = \sqrt{(u_{r,Q_L,meas})^2 + (u_{r,Q_L,mod})^2 + (u_{r,Q_L,meteo})^2} = 4.5\%$$

The calculated uncertainty, expressed as expanded uncertainty according to the usual metrological practice, is in the order of 9% for a confidence level of 95% and a coverage coefficient of $k=2$.

From the analysis of the results, it can be stated that the final uncertainty is mainly affected by the evident difficulty on the prediction of the meteorological conditions which might occur during the period the system would operate, as well as by the weakness of the model to accurately explain the experimental data. To a lesser degree, the final uncertainty presents dependence on the metrological level of the used measuring instruments, at least as far as this level remains within the limits indicated by the valid international standard.

4.3. The DST method

4.3.1 Test method and measurement model

Unlike the CSTG method, the DST method uses a dynamic energy model for the SDHWS. The testing procedure anticipates the implementation of specific operation scenarios for the system (test sequences). By the implementation of these sequences, measurements of climatic data, as well as of characteristic temperatures on the inlet and outlet of the solar tank are performed (CEN, 2006; ISO, 2007). Moreover, by the end of each sequence, a draw-off of the collected thermal energy which has remained in the tank takes place. On a first phase, some preparation of the primary source data takes place, mainly including low pass filtering for the elimination of the experimental noise. On a second phase, the elaborated experimental data are used for the identification of the coefficients of the energy model (Figure 4.8). On a third phase, the fitted model is used for the prediction of the expected energy output for specific conditions (Typical Meteorological Years of specific sites).

In the following paragraphs, the usual case of a typical solar-only SDHWS which has been installed outdoors. The experimental data recorded through the testing are the following:

- T_a : Ambient temperature (collector and store)
- G : Hemispherical irradiance on the collector plane
- T_{cw} : Water temperature at the store inlet (mains-water temperature)
- T_s : Store outlet temperature
- \dot{V}_s : Volumetric draw-off rate

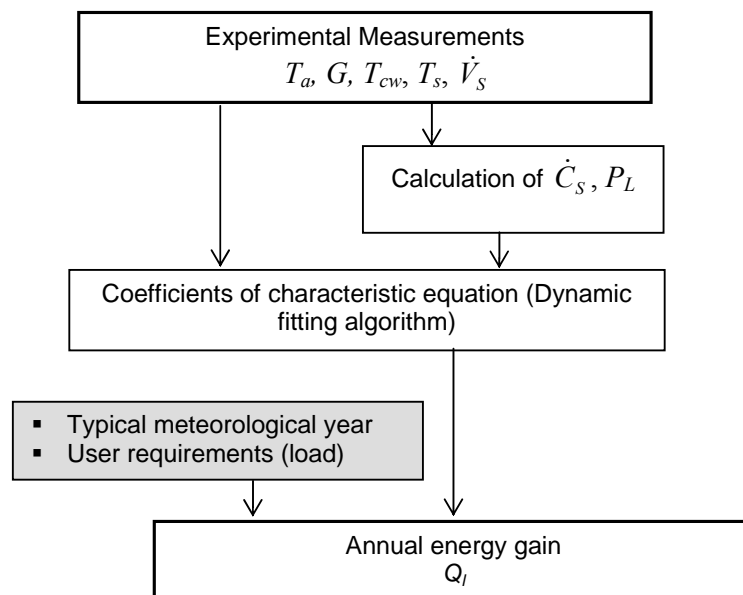


Figure 4.7. Flow-chart of actions required for the expected energy output calculation (DST method)

According to the procedures anticipated by the Standard, from the above mentioned experimental data the following quantities are calculated:

- $\dot{C}_S = c_w \rho_w \dot{V}_S$: Load capacitance rate through the store
- $P_L = \dot{C}_S (T_s - T_{cw})$: Draw-off load power

The problem with the DST method is that the required calculations are performed, at all stages, on the basis of a software which is available only in executable files form, while the implemented by these files algorithm is not available. Thus, the energy model used is not exactly known, neither the specific steps for the processing of the performance data. Moreover, the respective software is available only in DOS environment; this significantly limits the capability for the automation of the required calculations, and respectively the implementation of calculation-consuming approaches as the Monte-Carlo simulation.

The above mentioned problems are also barriers to the investigation for the potential errors of the method, and make difficult the assessment of the uncertainty characterizing the final result. In following sections, an assessment of the uncertainty components is performed, noting though that the whole process can be proven to be quite time consuming, given the difficulties discussed above.

In the case of the DST method, the basic assumptions which have been adopted in the case of the CSTG method are also valid (paragraph 4.2.3.1). These assumptions mainly concern:

- the need for the requirements of the Standard regarding the implementation of the test method to be satisfied;
- the fact that the calculated uncertainties concern the expected energy output of a SDHWS identical to the tested one and that the experience shows that in the case of the same production line, differences between products are quite limited and do not constitute a remarkable source of uncertainty;
- the assumption that the energy accumulated in the solar tank is considered to be drawn-off by the end of each day of the year, as anticipated in the standard (CEN, 2006);

Concerning the proposed methodological approach, the calculation of the uncertainty characterizing the expected annual energy output is implemented on five distinct steps, each of them corresponding to specific activities anticipated by the standardized testing method:

- I. Initially, the uncertainties related to the recorded values of the quantities T_a , G , T_{cw} , T_s , \dot{V}_S are estimated, on the basis of the metrological characteristics of the measuring equipment.
- II. The uncertainty component related to the potential errors of the measurement devices is estimated on the basis of the conclusions of a sensitivity analysis (section 4.3.2).
- III. The uncertainty component related to the imperfections of the energy model, i.e. its inability to explain precisely the experimental data gathered during the test, is estimated as discussed in section 4.3.3.
- IV. The uncertainty related to the fact that the meteorological conditions which may occur during the actual operation of the SDHWS system cannot be precisely known, is estimated as described in section 4.3.4.
- V. Finally, the combined uncertainty characterizing the final result is computed as the square root of the algebraic sum of the individual variance contributions (BIPM et al., 2008a).

All calculations have been performed in a MATLAB environment. Given that the procedure

used in the paper is complex and a large number of data is involved, special treatment regarding the checking of the validity of calculations has been given.

The presented methodological approach, has been implemented for the case of a typical domestic hot water system, with a collector aperture surface $A_c=3.76 \text{ m}^2$ and solar tank volume $V_s=191 \text{ l}$. As in the case of CSTG method, the system has been tested through the strict implementation of the standard requirements regarding the testing procedure, as well as the metrological quality of the calibrated sensors used.

4.3.2. Uncertainty component related to the errors of the sensors

As in the case of the CSTG method, given the non-algebraic character of the expected energy output calculation procedure, the most suitable method for the assessment of the specific uncertainty component would be this of the Monte-Carlo simulation (4.2.3.3). The inability though to automate the relevant calculations, due to the form of the involved software (protected MSDOS executables), makes this choice prohibitive. Given also the inability to implement the law of error propagation, the only realistic solution would be the exploitation of appropriate sensitivity studies, aiming at the quantification of the effect of potential measurement errors on the final result.

This analysis is based on the investigation of the variation of the final result (annual energy gain Q_l), as a function of the slight variations in the value of each measured parameter. For this reason, a slight variation dA is inserted in the experimental values of the parameter A , which have been recorded during the tests. Following, a new value Q'_l of the annual energy gain is calculated, as well as the difference of the new value to the initially calculated one, $dQ_l = Q'_l - Q_l$. The sensitivity factor for the specific parameter can be estimated by the relation:

$$\frac{\partial Q_l}{\partial A} \approx \frac{dQ_l}{dA} \quad (4.16)$$

Table 4.3: Sensitivity coefficients

Quantity	Mains-water temperature	Store outlet temperature	Ambient temperature	Solar irradiance	Draw-off flow rate
Sensitivity coefficient	$\frac{\partial Q_l}{\partial T_{CW}} = -38$ [kWh m ⁻² K ⁻¹]	$\frac{\partial Q_l}{\partial T_s} = +65$ [kWh m ⁻² K ⁻¹]	$\frac{\partial Q_l}{\partial T_a} = -18$ [kWh m ⁻² K ⁻¹]	$\frac{\partial Q_l}{\partial G} = -0.64$ [kWhm ⁻² /W m ⁻²]	$\frac{\partial Q_l}{\partial \dot{V}_s} = +1.43$ [kWh m ⁻² /(lt h ⁻¹)]

In order for the specific estimation to be reliable, the relevant investigation has been repeated for 4 different values of dA , evenly distributed around the experimentally measured value. Moreover, the investigation has been repeated for all parameters, the measured value of which influences the final result. Even though this approach can be implemented for any draw-off

profile, for the reasons explained in section (4.2.1) the presented results concern the case of a system subject to the draw off of three tank volumes by the end of each day. The results are presented in figures 4.8-4.12, for each one of the measured parameters. The respective sensitivity coefficients are presented in table 4.3.

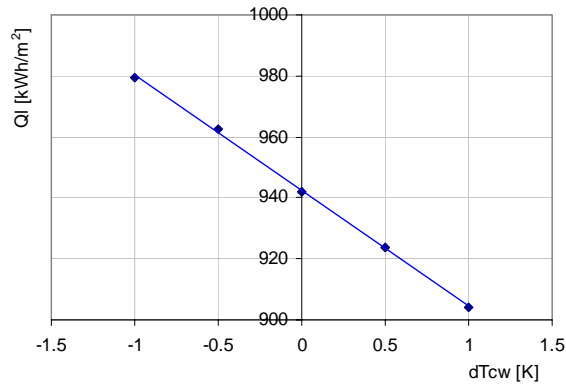


Fig. 4.8. Variation of annual energy gain with small changes in mains-water temperature

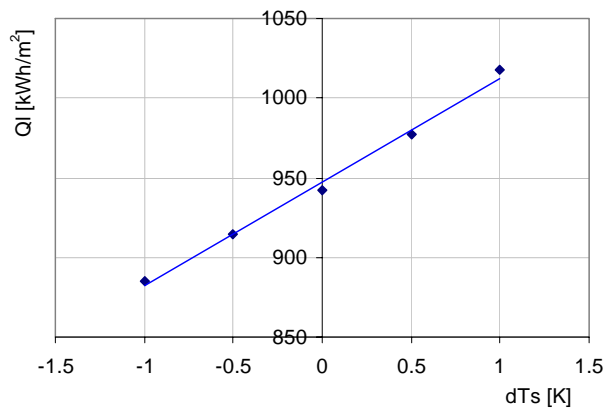


Fig. 4.9. Variation of annual energy gain with small changes in store outlet temperature

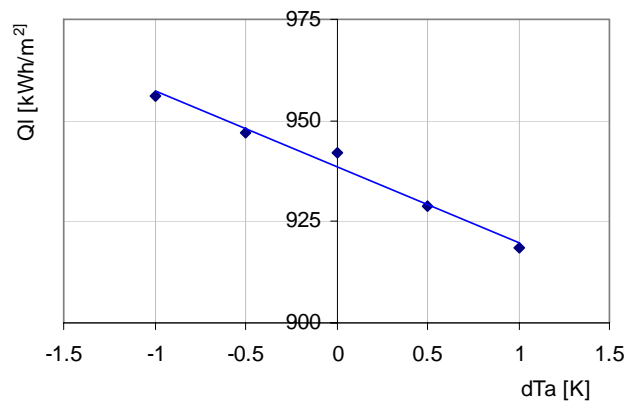


Fig. 4.10. Variation of annual energy gain with small changes in ambient temperature

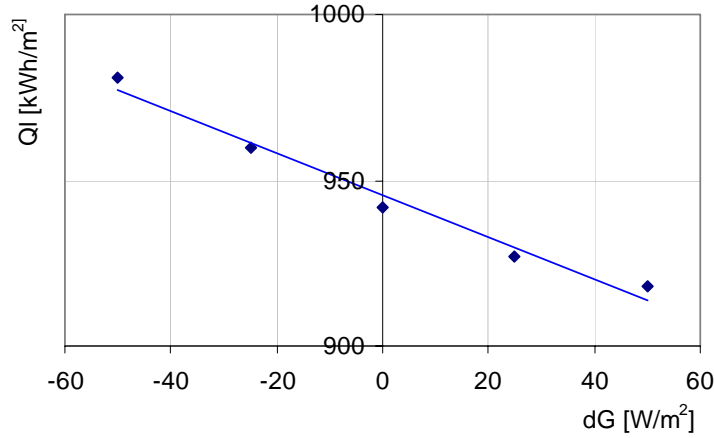


Fig. 4.11. Variation of annual energy gain with small changes in incident solar radiation

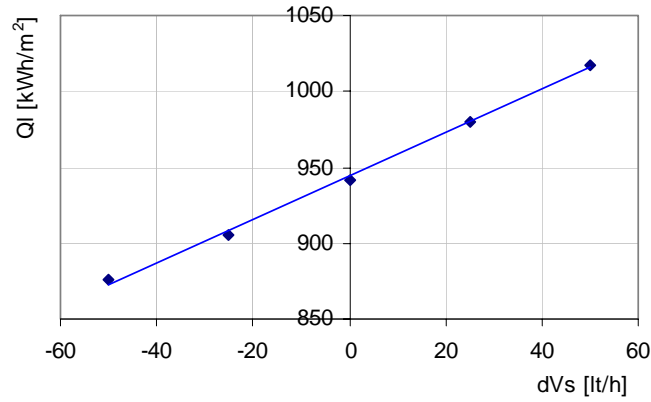


Fig. 4.12. Variation of annual energy gain with small changes in volumetric draw-off rate

The uncertainty component related to the errors of the sensors can therefore be estimated by the relation:

$$u_{Q_l, meas} \approx \sqrt{\left(\frac{\partial Q_l}{\partial T_{CW}} u_{T_{CW}}\right)^2 + \left(\frac{\partial Q_l}{\partial T_s} u_{T_s}\right)^2 + \left(\frac{\partial Q_l}{\partial T_a} u_{T_a}\right)^2 + \left(\frac{\partial Q_l}{\partial T_G} u_G\right)^2 + \left(\frac{\partial Q_l}{\partial T_{V_s}} u_{V_s}\right)^2} \quad (4.17)$$

The metrological quality of the measurement devices which have been used, being compatible with the requirements of the respective standard, can be formulated on terms of standard uncertainty as follows:

- For the inlet and outlet temperature of the fluid (T_{CW} and T_s): $u_{T_{CW}} = u_{T_s} = 0.1$ K.
- For the ambient temperature: $u_{T_a} = 0.29$ K.
- For the flow rate of the thermal medium: $u_{V_s} = 3.5$ lt/h.

- For the incident solar radiation (by using a first class pyranometer): $u_G = 25 \text{ Wm}^{-2}$.

As in the case of the CSTG method, if the metrological requirements are formulated on accuracy terms, the calculation of standard uncertainty u_A of a quantity A is based on the consideration of an orthogonal probability distribution; the uncertainty is correlated to the respective accuracy a_A through equation 4.15.

From equation 4.17 and the values of sensitivity coefficients presented in table 4.3, an uncertainty value of $u_{Q_t, meas} = 19 \text{ kWh m}^{-2}$ or, in terms of relative uncertainty, $u_{r, Q_t, meas} = 2.0\%$, can be concluded for the respective test.

4.3.3. Imperfections of the energy model and related uncertainty

As it has already been pointed out, the difficulty related to the calculation of uncertainty in the DST method concerns the exact energy model, as well as the algorithm for the primary data elaboration, not being accessible to the user. For this reason it is difficult to calculate analytically the component of uncertainty related to the imperfections of the energy model, which expresses the degree the model can explain the experimental data.

An estimation though of this uncertainty component is provided by the software of the expected annual energy gain calculation itself (*Long Term Performance Prediction* module of the software). According to the documentation accompanying the software, the relevant calculations use the cross correlation matrix of the energy model coefficients which have been iteratively determined through the fitting procedure (*Parameter Identification* module of the software) (Spirkl, 1997).

It is obvious that in order to consider the fitting typical error as a realistic assessment of the related to the imperfections of the energy model errors, the tests should have included a potentially wide and representative range of operation conditions for the system.

For the examined case, a value for the relative uncertainty component $u_{r, Q_t, mod} = 1.1\%$ has been calculated.

4.3.4. Uncertainty component related to the variation of the meteorological conditions

Similarly to the case of the CSTG method, the uncertainty component related to the variation of the meteorological conditions has been estimated according to the approach presented in 4.2.3.5. The annual energy output for 20 consecutive years has been calculated, by using the meteorological data of the period from year 1989 to year 2008, for the geographical area of Athens (courtesy of the Institute of Environmental Research and Sustainable Development - National Observatory of Athens). The calculation has been implemented on the basis of the calculation method anticipated in Standard EN 12976-2 (DST method).

Typical results for a draw-off volume $V_d = 3V_c$ are presented in Figure 4.13 in terms of deviation from the 20-years mean values, while the related uncertainty component, calculated according to Equation 4.16, is presented in Figure 4.14 for different draw-off volumes. The component of relative uncertainty is estimated to be in the order of $u_{r, Q_t, meteo} = 3.7\%$, which is very close to the value calculated for the CSTG method.

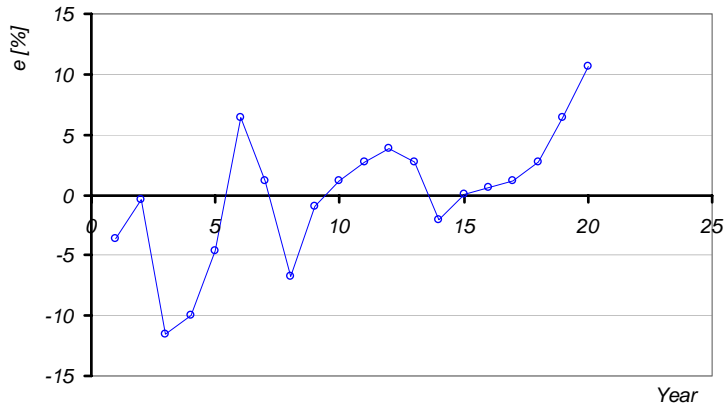


Figure 4.13: Deviations of the annual energy output for 20 different meteorological years from the 20-year mean value (DST method)

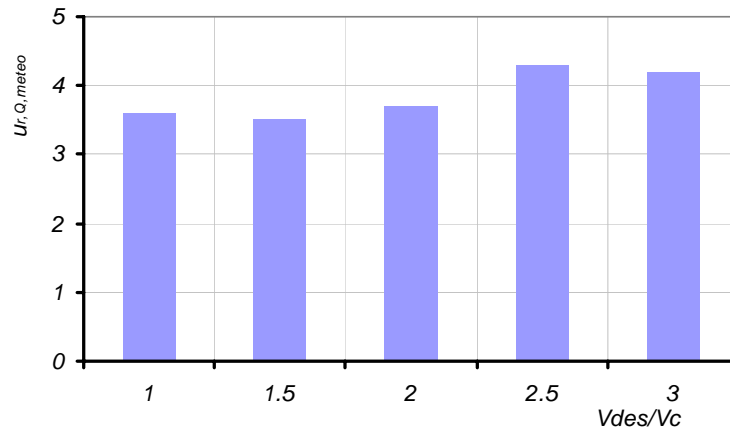


Figure 4.14: Relative uncertainty of the annual energy output for 20 different meteorological years with regard to the hot water volume drawn-off (DST method)

4.3.4. Total combined uncertainty

The combined standard uncertainty associated with the values of the annual expected energy output is calculated through the combination of the individual components:

$$u_{r,Q_i} = \sqrt{(u_{r,Q_i,meas})^2 + (u_{r,Q_i,mod})^2 + (u_{r,Q_i,meteo})^2} = 4.3\%$$

As in the case of the CSTG method, the calculated uncertainty, expressed as expanded uncertainty according to the usual metrological practice, is in the order of 8.6% for a confidence level of 95% and a coverage coefficient of $k=2$.

Similarly to the case of the CSTG method, it can be concluded that the final uncertainty is mainly affected by the evident difficulty on the prediction of the meteorological conditions which might occur during the period the system would operate. In contrast to the CSTG method though, the metrological level of the used measuring instruments proves to significantly influence the final uncertainty budget, at least compared to the respective degree of influence of the weakness of the energy model to accurately explain the experimental data.

5. Conclusions

The scope of this work is the systematic discussion of the problems related to the estimation of uncertainty characterizing the results of solar thermal collectors and systems testing. Within this context, a complete methodological approach has been developed; the analysis has included specific examples referring to the implementation of the method in actual testing data.

In the case of solar thermal collectors testing, the rather simple procedure for the testing and - mainly - for the elaboration of the results, has made possible the formulation of a straightforward proposal regarding the steps to be implemented and the required calculations. The proposed method allows not only the realistic assessment of the uncertainty characterizing the result, but also the suitability of the energy model used.

In the case of solar systems, the involved calculations are more complex, thus the problem to be solved is more difficult. In the case of the CSTG method, the use of the Monte-Carlo simulation techniques allows the assessment of the effect of the measurement sensor errors, through the implementation of propagation of distributions concept.

In the case of the DST method additional difficulties appear, related to the fact that the energy model as well as the algorithm for the elaboration of the experimental data are not known. For this reason, sensitivity analysis techniques have been used, allowing the approximate assessment of the effect of the experimental errors.

Through the interpretation of the results presented in this study, one should bear in mind the following:

- As pointed out through the analysis, there does not exist a unique path for the calculation of uncertainties. The proposed approaches should be treated as one amongst the potential alternatives, and should not be considered as the only solution. Every laboratory is responsible for the elaboration of its own approach, provided that this approach is compatible with the commonly accepted metrological practice.
- The uncertainty characterizing a quantitative testing result depends on the specific conditions of the test performed, as well as on the characteristics of the device under test. From this point of view, the uncertainty figures presented concern only the tests which have been included as examples in this study. However, the experience gained by our Laboratory through the repeated implementation of the proposed methodologies in actual testing cases, shows that the presented uncertainty values can be considered representative for the type of products they refer to. A systematic investigation of the resulting uncertainties level by other laboratories, including a wider range of solar products, as well as the exchange of experience, could allow the formulation of a more realistic picture for the discussed topic. In such a case, it would be feasible to propose uncertainty figures representative of each test.

- An interesting finding of the whole investigation is that the requirements of the standards, regarding the metrological quality of the equipment used, are in general satisfactory, as they do not seem to significantly burden the uncertainty budget. The uncertainties related with the quality of the energy model, and its ability to predict the behavior of the product throughout a wide range of operation conditions relevant to the ones the system would face in actual operation, prove to be more significant.

Especially for solar thermal systems, it should be stressed that the expected annual energy gain is significantly influenced by the meteorological conditions valid during the actual operation of the system. The variability of these conditions seems to represent the most significant source of uncertainty, at least as regards the future user of the system. However, if it is clarified that the proposed value for the expected annual energy gain concerns the adopted Typical Meteorological Year, the respective uncertainty coefficient can be ignored.

References

Belessiotis, V., Mathioulakis, E., Papanicolaou, E., 2010. Theoretical formulation and experimental validation of the input–output modeling approach for large solar thermal systems. *Solar Energy* 84(2), 245–255.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, OIML, 2008a. Evaluation of Measurement Data - Guide to the Expression of Uncertainty in Measurement. <http://www.bipm.org/utis/common/documents/jcgm/JCGM_100_2008_E.pdf>.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, OIML, 2008b. Evaluation of Measurement Data – Supplement 1 to the ‘Guide to the Expression of Uncertainty in Measurement - Propagation of Distributions Using a Monte Carlo Method. <www.bipm.org/utis/common/documents/jcgm/JCGM_101_2008_E.pdf>.

Bourges, B., Rabl, A., Carvalho, M.J., Collares-Pereira, M., 1991. Accuracy of the European solar water heater test procedure, Part 2: Long-term performance prediction. *Solar Energy* 47(1), 17-25.

Burhenne, E., Jacob, D., Henze, G., 2010. Uncertainty analysis in building simulation with Monte-Carlo techniques, 4th National Conference of IBPSA-USA, New York, August 11 – 13.

Cecchi G. C., *Meas. Sci. Technol.*, 1991, **2**, 1127-1128; Erratum, 1993, **4**, 906.

CEN, 2006a. European Standard EN12975-2:2006. Thermal solar systems and components - Solar collectors – Part 2: Test methods. CEN, Brussels.

CEN, 2006b. European Standard EN12976-2:2006. Thermal solar systems and components - Solar collectors – Part 2: Test methods. CEN, Brussels.

Cox, M.G., Siebert, B.R.L., 2006. The use of a Monte Carlo method for evaluating uncertainty and expanded uncertainty. *Metrologia* 43, S178–S188.

Gazela, M., Mathioulakis, E., 2001. A new method for typical weather data selection to evaluate long term performance of solar energy systems. *Solar Energy* 70(4), 339-348.

ISO, 1995. ISO 9459-2, Solar heating - Domestic water heating systems - Part 2: Outdoor test methods for system performance characterization and yearly performance prediction of solar-only systems. ISO Ed., Geneva.

ISO, 2007. ISO 9459-5, Solar heating -- Domestic water heating systems -- Part 5: System performance characterization by means of whole-system tests and computer simulation. ISO Ed., Geneva.

Li, Y-C., Lu, S-M., 2005. Uncertainty evaluation of a solar collector testing system in accordance with ISO 9806-1. *Energy* 30(13), 2447-2452.

Lira, I., 2000, Curve adjustment by the least-square method, *Metrologia*, 37, pp. 677-682

Lira, I., 2002. Evaluating the measurement uncertainty. IoP ed., Bristol.

Lira, I., Elster, C., Wöger, W., Cox, M.G., 2009. Derivation of an output PDF from Bayes’ theorem and the principle of maximum entropy, in: Pavese, F., Bär, M., Linares, J.M., Perruchet, C., Zhang, N.F. (Eds.), *Advanced Mathematical & Computational Tools in Metrology VIII, Series on Advances in Mathematics for Applied Sciences vol. 78*, World Scientific New Jersey, p.p. 213-218.

- Mathioulakis, E., Voropoulos, K., Belessiotis, V., 1999. Assessment of uncertainty in solar collector modeling and testing. *Solar Energy* 66(5), 337-347.
- Matsumoto, M., Nishimura, T., 1998. Mersenne Twister: A 623-dimensionally equidistributed uniform pseudorandom number generator. *ACM Transactions on Modeling and Computer Simulation* 8, 3-30.
- Müller-Schöll, C and Frei, U., Uncertainty Analyses in Solar Collector Measurement, Proceedings of Eurosun 2000, Copenhagen
- Press W., Teukolsky S.A., Vetterling W.T. and Flannery B.P. (1996), *Numerical recipes*, 2nd edn., Cambridge University Press, Oxford.
- Sabatelli V., Marano D., Braccio G. and Sharma V. K., Efficiency test of solar collectors: uncertainty in the estimation of regression parameters and sensitivity analysis, *Energy Conversion and Management*, Vol. 43(17), 2002, pp. 2287-2295
- Spirkl W. ISS (InSitu Scientific Software). Dynamic System Testing Program Manual (Version 2,7). Kriegerstr 23d, Germering, 1997.
- WMO, 2008. Guide to Meteorological Instruments and Methods of Observation (7th ed.). World Meteorological Organization, Geneva.
- Wubbeler, G., Krystek, M., Elster, C., 2008. Evaluation of measurement uncertainty and its numerical calculation by a Monte Carlo method. *Measurement Science and Technology* 19(8), 084009.