PERGAMON

Energy Conversion and Management 43 (2002) 2287–2295

ENERGY CONVERSION & MANAGEMENT

www.elsevier.com/locate/enconman

Efficiency test of solar collectors: uncertainty in the estimation of regression parameters and sensitivity analysis

V. Sabatelli, D. Marano, G. Braccio, V.K. Sharma *,[1]

ENEA CR Trisaia, Solar Energy Laboratory, 75026, Rotondella (MT), Italy

Received 22 June 2001; accepted 2 November 2001

Abstract

The results obtained from efficiency tests conducted on a flat plate solar collector, according to the ISO 9806/1 test procedure, have been used to determine the uncertainty in the curve fitting parameters. The said standard, though requiring certain levels of accuracy in the measuring process, doesnot provide any method to determine the uncertainty of the efficiency curve parameters. The methodology used in the present paper (not provided by the ISO standard) allows solving the above mentioned problem and evaluating not only the parameters and their uncertainties but also the reliability of the test procedure and its goodness toward fitness. In order to evaluate the effects of measurement errors on the values of the uncertainty in estimated parameters, a sensitivity analysis has also been conducted. Strong dependence of some uncertainties, involving a larger accuracy level in the estimation of the measured parameters, is a clear indication of the present investigations. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Collector efficiency test; Chi-square fitting; Uncertainty in estimated parameters; Sensitivity analysis

1. Introduction

ISO standards are mainly used for determination of the collector efficiency by means of a least squares fit performed on the measured data. It is, however, to be noted that to use such standard, it is essential to have a certain level of accuracy, especially for the sensors and instrumentation, but the fact remains that no prescribed methodology is available to determine the uncertainty of the efficiency curve parameters.

---

* Corresponding author. Address: ENEA Research Centre Trisaia, AMB-TEIN-RIF Unit, 75025, Policoro (MT), Italy. Tel.: +39-835-974270; fax: +39-835-974210.

E-mail address: sharma@trisaia.enea.it (V.K. Sharma).

[1] Visiting Scientist.

PII: S0196-8904(01)00180-7

Nomenclature

A	collector aperture (m^2)
T_{in}	inlet temperature (°C)
T_{out}	outlet temperature (°C)
ΔT	temperature difference (°C) $\Delta T = T_{out} - T_{in}$
Γ	mass flow rate ($kg\,s^{-1}$)
G	global solar irradiance ($W\,m^{-2}$)
T_a	ambient temperature (°C)
T_m	mean temperature (°C), equal to $(T_{in} + T_{out})/2$
T^*	reduced temperature ($m^2\,W^{-1}\,K$), equal to $(T_m - T_a)/G$
$\sigma_{A,x}$	Type A standard uncertainty
u_B	Type B standard uncertainty

In order to solve the above mentioned problem, an attempt has been made to deal with the problem using both the results available in the literature [1,2] and experimental data collected in our laboratory. A detailed sensitivity analysis has been conducted to determine the influence of the uncertainties in the measured parameters on the uncertainty level that can be assigned to coefficients of the efficiency curve.

The present work has been done by a group of researchers from the Solar Collector and Overall System Test Laboratory at the ENEA (Agency for New Technology, Energy and Environment) Research Centre, Trisaia, in Southern Italy. The laboratory performs efficiency tests both on glazed and unglazed solar collectors, according to ISO standards 9806/1 and 3. Systems performance tests are conducted using ISO standard 9459/2 [3–5]. The experimental data collected over a period of nearly two years has been used. Over this period, about 30 collectors, made in Italy and/or abroad, have been tested. The sensors, instrumentation and control system used permits an accuracy level much higher than the one requested by the ISO standards.

At present, though the test facilities are available to work with collectors in a fixed position (facing the equator) in a semi-automatic way, steps have already been taken to make the system fully automatic while working with sun following solar collectors.

The sensors and instrumentation employed, along with their accuracy, a brief resume of uncertainty theory and curve fitting and, finally, a sensitivity analysis performed and the results obtained, are discussed in the present paper.

2. Sensors and instrumentation used in the laboratory

A brief description of the solar collector test facilities, accuracy of the sensors and instrumentation employed and their comparison with recommended ISO standard values is presented in this section.

The entire necessary infrastructure for implementation of solar collector efficiency tests, as per ISO standard, is available in the laboratory. The solar collector test facilities, a closed loop with

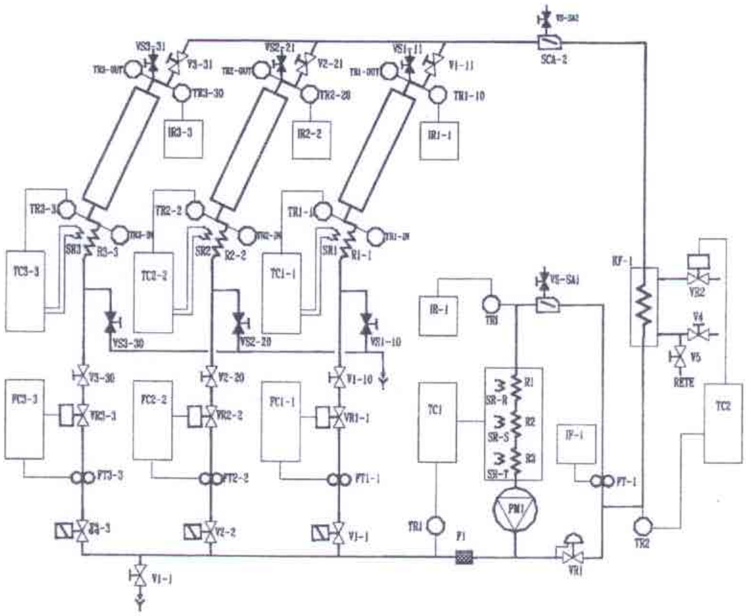

Fig. 1. Block diagram of solar collector test facilities.

heat exchanger configuration (Fig. 1), allow the simultaneous efficiency testing of three collectors arranged in parallel. The equipment consists of a primary temperature control system, helped by secondary temperature controls (three in number), located near the collectors under test that allow adjustment of the inlet temperature within the range required by the ISO standard. The operation is accomplished through digital proportional, integral and derivative control, provided by *self-tuning* and controlled through a programmable logic controller. Furthermore, a mass flow rate control system maintains the inlet water flow for each collector under test to be stable. The adjustment is accomplished in the same manner as that of temperature control. Also, the laboratory is equipped with a weather station for measurement of the meteorological data.

A PC based data acquisition system, through a *user-friendly* graphic interface, allows control and data collection of the thermo-hydraulic and meteorological parameters. A general purpose data acquisition card (NI-DAQ PCI-MIO-16XE-50) provides control of the whole acquisition system and the A/D conversion of the acquired analogical signals through real time sampling. An external signal conditioning system, driven by the same card, provides the current supply of the Pt100 sensors, amplifying and filtering the input signals.

Concerning the sensors used in the laboratory, generally two types of parameters, i.e. *thermo-hydraulic* (inlet temperature, temperature difference between inlet and outlet and flow rate) related to the collector to be tested and *meteorological* (global and diffuse irradiance, ambient temperature and wind speed) related to the environmental conditions, need to be measured.

To achieve an accuracy of 0.05 °C (for the inlet temperature) and 0.07 °C (for the difference between the inlet and outlet temperatures), individually calibrated *Pt100 sensors* were used. In order to avoid a separate measurement of density while determining the mass flow rate, a *Coriolis Meter* was used. The flow rate was measured within an accuracy of 0.5%.

Table 1
Laboratory accuracy levels compared with those required by ISO standard

	Accuracy	
	Required by ISO 9806/1	Measured in laboratory
Inlet temperature	±0.1 °C	±0.05 °C
Temperature difference	±0.1 °C	±0.07 °C
Flow rate	±1.0% (of reading)	±0.5% (of reading)
Solar irradiance	Pyranometer of Class I (according to ISO 9060)	Pyranometer of Class I (according to ISO 9060)
Ambient temperature	±0.5 °C	±0.2 °C
Wind speed	$\pm 0.5\ \mathrm{m\,s^{-1}}$	$\pm 0.5\ \mathrm{m\,s^{-1}}$

The ambient temperature was measured (within an accuracy of 0.2 °C) in a well ventilated but shaded shelter (from direct and reflected solar radiation) using a Pt100 sensor. The irradiance was measured using a first class *pyranometer* (according to ISO 9060), whereas for wind velocity measurement (with $0.5\ \mathrm{m\,s^{-1}}$ accuracy), a *taco-anemometer* was employed.

The above mentioned accuracy values are listed and compared with those required by the ISO standard in Table 1.

The collector efficiency, using a 3-parameters model, is expressed as:

$$\eta = \eta_0 + a_1 T^* + a_2 G T^{*2} \tag{1}$$

where

$$T^* = \frac{\left(T_{\mathrm{in}} + \frac{\Delta T}{2}\right) - T_{\mathrm{a}}}{G} \quad \text{and} \quad \Delta T = T_{\mathrm{out}} - T_{\mathrm{in}} \tag{2}$$

The ISO standard requires use of the least squares method in order to determine the three parameters, i.e. η_0, a_1 and a_2. It is, however, to be noted that the above procedure is correct only with the assumptions that the uncertainty in efficiency measurement is the same for all the data collected and errors affecting T^* and GT^{*2} are negligible. Moreover, this cannot be used to evaluate the uncertainty affecting η_0, a_1 and a_2, which obviously, are correlated to the uncertainties in the measured parameters. In order to solve this problem, an alternate algorithm has been discussed in the following paragraph.

3. Uncertainties in measured data

3.1. Fundamentals and errors evaluation

The uncertainty of measurement is defined as the parameter associated with the result of a measurement characterising the dispersion of the values that could reasonably be attributed to the measurand. The object of a measurement is the estimation of the "true value" of a measurand. The true value is the ideal result that one could obtain only by means of a perfect measurement. A real measurement is affected by errors. Even, if all the errors could be evaluated and corrected, there still remains an uncertainty about the result of the measurement that should be considered only as an estimation of the measurand.

When a measurement is repeated under the same conditions, one can observe a scatter of the measured values. This may be attributed to the statistical effects caused by variations in the ambient conditions. The related uncertainty, called "Type A" uncertainty, from N observations x_j (with the arithmetic mean $\bar{x}$) related to a quantity X, is given by:

$$\sigma_{\mathrm{A},X} = \left(\frac{\sum_j (x_j - \bar{x})^2}{N(N-1)} \right)^{0.5} \tag{3}$$

As is evident from Eq. (3), the Type A uncertainty depends upon the specific conditions of the measurement and, thus, can be reduced by increasing the number of measurements.

"Type B" uncertainty, being dependent upon the measuring instrument, cannot be reduced by augmenting the number of measurements. Generally, sensor specifications furnish the "accuracy" a of an instrument, which is the maximum deviation from the true value. It is assumed that all values inside the interval $2a$ are equally probable. In this case, it is possible to demonstrate that the uncertainty can be obtained from the following equation:

$$u_{\mathrm{B}} = \frac{a}{\sqrt{3}} \tag{4}$$

If a measurand y depends on a number of measured parameters x_i, to which uncertainties are associated, then the uncertainty contribution $u_i(y)$ from x_i to y is given by:

$$u_i(y) = c_i u_{\mathrm{B}}(x_i) \tag{5}$$

where c_i, the coefficient of sensibility, is given by:

$$c_i = \frac{\partial y}{\partial x_i} \tag{6}$$

For uncorrelated parameters, the global uncertainty is obtained as root sum of squares:

$$u(y) = \sqrt{\sum_i \left[\left(\frac{\partial y}{\partial x_i} \right) u(x_i) \right]^2} \tag{7}$$

In order to evaluate the global uncertainty of a measurand, it is necessary to consider all the possible error sources. This involves an evaluation not only of the sensor uncertainty but also of the whole data acquisition chain. In view of this fact, the uncertainties in the measurement and characteristic at Trisaia Laboratory are shown in Table 2.

3.2. Curve fitting and uncertainty in estimated regression parameters

The parameters η_0, a_1 and a_2 that best fit the collector efficiency model, Eq. (1) are the solution of a multiple linear regression problem [6]. In general, it is essential to find the parameters that are the maximum likelihood estimators of M coefficients that best fit N linear combinations of *basis functions* $X_1(x_i), X_2(x_i), \ldots, X_M(x_i)$. The measure of agreement between the experimental data and

Table 2
Typical measurement uncertainties (Trisaia Laboratory)

	Uncertainty
Inlet temperature	±0.025 °C
Temperature difference	±0.036 °C
Flow rate	±0.25% (of reading)
Solar irradiance	±1.6% (of reading)
Ambient temperature	±0.12 °C
Wind speed	$\pm 0.3\ \mathrm{m\,s^{-1}}$

the model, with this particular choice of parameters, is accomplished through a suitable *merit function*, which is arranged so that the best fit occurs in a minimum of the function.

If the model that fits the N data points (x_i, y_i) is:

$$y(x_i, a_1, a_2, \dots, a_M) = a_0 X_0(x_i) + a_1 X_1(x_i) + \cdots + a_M X_M(x_i) \tag{8}$$

and the experimental points y_i, characterized by errors with normal distribution around the true value $y(x_i)$, with a standard deviation σ equal for all points, the merit function is:

$$\sum_{i=1}^{N} [y_i - y(x_i, a_1, a_2, \dots, a_M)]^2 = \sum_{i=1}^{N} \left[y_i - \sum_{k=1}^{M} a_k X_k(x_i) \right]^2 \tag{9}$$

the same as the *least squares method*.

In reality, the uncertainties of each experimental point are different. This involves changing the merit function, which becomes the χ^2 function:

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{y_i - y(x_i, a_1, a_2, \dots, a_M)}{\sigma_i} \right]^2 = \sum_{i=1}^{N} \left[\frac{y_i - \sum_{j=1}^{M} a_j X_j(x_i)}{\sigma_i} \right]^2 = |\mathbf{B} - \mathbf{A} \cdot \boldsymbol{\alpha}|^2 \tag{10}$$

where $A_{i,j} = X_j(x_i)/\sigma_i$ are the elements of the *design matrix* of the fitting problem, $B_i = y_i/\sigma_i$ is a vector of length N, whose elements are the values to be fitted, weighted by their uncertainties, and α_j is the vector of parameters.

There are different ways to minimize the χ^2 function. One way is to set the partial derivatives of χ^2 to zero with respect to the parameters $a_1, a_2, \dots, a_M$. This allows achieving some *normal equation* that can be written as follow:

$$\left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right) \cdot \boldsymbol{\alpha} = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{B} \tag{11}$$

The solution of this matrix equation provides the best fit parameters as well as a $M \times M$ matrix, $C = \left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right)^{-1}$, where the diagonal elements are the squared uncertainties of the regression parameters and the off diagonal elements are the co-variances between them. If the experimental data is subjected to measurement errors not only in the y_i's, but also in the x_i's (our case), each element of the χ^2 merit function must be weighted by uncertainties, which depend on the fitted parameters. Consequently, the minimization of Eq. (10) cannot be solved with a closed regression operation but has to be solved with an iterative algorithm for minimization of the non-linear function. All that is said in the above paragraph has been applied to the results of collectors tested

Table 3
Estimated collector parameters and their standard uncertainties

	η_0	a_1 ($\mathrm{W\,m^{-2}\,K^{-1}}$)	a_2 ($\mathrm{W\,m^{-2}\,K^{-2}}$)
Parameter value	0.781	−4.2	−0.012
Uncertainty	0.007	0.4	0.006

in the Trisaia Laboratory. An example of estimated collector parameters, together with their standard uncertainties, is presented in Table 3.

These values have been determined by starting from "guess" parameters, retrieved by the usual least squares method, and using the Levenberg–Marquardt method to minimize the non-linear χ^2 function.

A number of experimental points used for this analysis with their standard error bars are shown in Fig. 2. Standard uncertainty ranges of the efficiency curve are also plotted in the same figure.

The χ^2 merit function, together with the $\nu = N - M$ degrees of freedom of the problem, also provides a quantitative indication of the goodness-of-fit for the specific model. In general, if the quantity:

$$Q(0.5\nu, 0.5\chi^2) = \frac{1}{\Gamma(0.5\nu)} \int_{0.5\chi^2}^{\infty} \mathrm{e}^{-t} t^{0.5\nu-1}\,\mathrm{d}t \tag{12}$$

is larger than 0.1, there is good agreement between the model and the data. If Q is larger than 0.001, the agreement is still acceptable but with caution. Instead, if Q is less than 0.001 the model, or the uncertainties estimated, can be doubtful. In our case, a value of $Q = 0.7$ underlines a very good agreement between the model and the experimental data.

3.3. Sensitivity analysis

In order to evaluate the effect of uncertainty in the experimental measurements on the values of the uncertainties in the estimated regression parameters, a sensitivity analysis has been conducted.

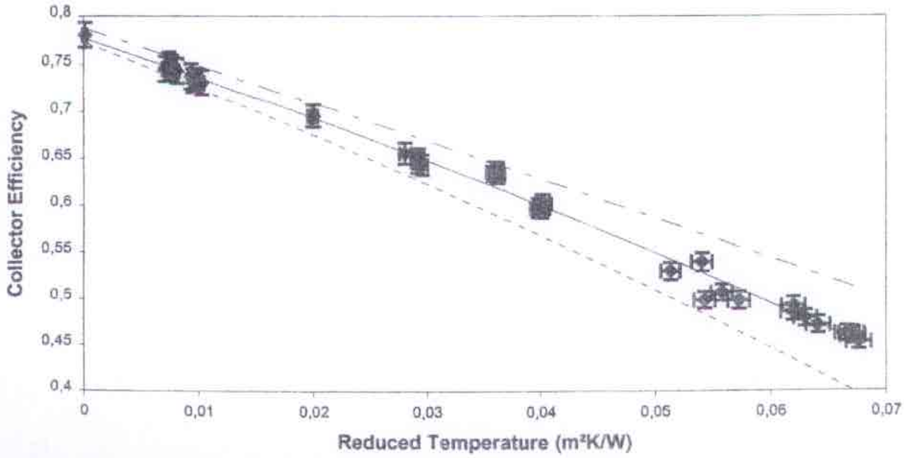

Fig. 2. Experimental points with standard uncertainty range of efficiency curve.

Table 4
Sensitivity analysis for uncertainties in estimated parameters

Parameter	Variation (%)	$\Delta(u_{\eta 0})$ (%)	$\Delta(u_{a1})$ (%)	$\Delta(u_{a2})$ (%)
$u_{T_{in}}$	±10	0.001 – 0.001	0.002 – 0.001	0.002 – 0.001
	±20	0.003 – 0.002	0.003 – 0.003	0.003 – 0.002
	±30	0.004 – 0.003	0.005 – 0.003	0.005 – 0.004
$u_{\Delta T}$	±10	1.0 – 0.9	1.1 – 1.0	1.1 – 1.0
	±20	2.0 – 1.7	2.3 – 1.9	2.4 – 2.0
	±30	3.1 – 2.4	3.6 – 2.8	3.7 – 2.8
u_{Γ}	±10	0.2 – 0.2	0.2 – 0.2	0.2 – 0.2
	±20	0.5 – 0.4	0.4 – 0.3	0.4 – 0.3
	±30	0.7 – 0.5	0.7 – 0.5	0.6 – 0.5
u_G	10	8.9 – 8.8	8.8 – 8.7	8.7 – 8.6
	±20	17.9 – 17.4	17.6 – 17.2	17.6 – 17.1
	±30	26.9 – 25.9	26.5 – 25.5	26.5 – 25.4
u_Ta	±10	0.03 – 0.02	0.03 – 0.03	0.03 – 0.03
	±20	0.06 – 0.05	0.07 – 0.05	0.07 – 0.06
	±30	0.09 – 0.07	0.11 – 0.08	0.11 – 0.08

For this purpose, all measurement errors of the individual parameters have been considered to be either higher or lower by certain percentages. The respective measurement errors have been changed one at a time, keeping the remaining ones constant. When a parameter is changed, a new set of collector parameters with their uncertainties is recalculated. The relative difference between this new set and the original results is computed.

It is evident from the results presented in Table 4 that the uncertainties in the estimated regression parameters are very sensitive to the measurement error of solar irradiance (a change of 30% involves variations upper to 25%), and therefore, solar irradiance should be acquired with a larger accuracy level. Small but appreciable sensitivity has been found in the uncertainty change relevant to temperature difference measurement, whereas no appreciable influences seem to be in the uncertainty variations of mass flow rate and inlet and ambient temperatures.

4. Conclusions

The results of a flat plate solar collector efficiency test have been examined. An estimation of the parameters with their uncertainties has been developed according to a methodology, based on well known statistical tools, which takes into account the global uncertainties of the measurement process (not only the uncertainties in efficiency measurement, different from measure to measure, but also the errors affecting T^* and GT^{*2} in the 3-parameters model). The sensitivity analysis performed has also emphasized a strong dependence of the values of the uncertainty in the estimated parameters from some measurement errors.

The following conclusions can be drawn, starting from these evaluations:

- The ISO standard requires the use of the least squares method in order to estimate the efficiency curve parameters without suggesting any methodology to evaluate the uncertainties in the es-

timated parameters. In this manner, not only no information about the reliability of curve fitting can be retrieved, but also it is difficult to make a comparison of the results obtained from different laboratories.
- The use of χ^2 fitting not only helps to solve this problem but also allows a check of the test procedure and the goodness-of-fit.
- The experimental results have emphasized that the uncertainties in the estimated regression parameters are mainly due to errors associated with the measurements of solar irradiance and temperature difference. More accurate measurement of such parameters would certainly be helpful to improve the whole uncertainty picture.

Acknowledgements

One of the authors (V. K. S) would like to express his sincere thanks to Prof. G. Furlan, Head, ICTP Programme for Training and Research in Italian Laboratories, for financial support and constant encouragement throughout the course of this work.

References

[1] Mathioulakis E, Voropoulos K, Belessiotis V. Solar energy 1999;66(5):337.
[2] Müller-Schöll C, Frei U. Uncertainty analysis in solar collector measurement, EuroSun 2000 Conference, 19–22 June, Copenhagen, Denmark.
[3] Test method for solar collector-Part 1: Thermal performance of glazed liquid heating collectors including pressure drop. International Standard ISO 9806/1, 1994.
[4] Test method for solar collector-Part 3: Thermal performance of unglazed liquid heating collectors (sensible heat transfer only) including pressure drop. International Standard ISO 9806/3, 1995.
[5] Solar heating—Domestic water heating system-Part 2: Outdoor test method for system performance characterization and yearly performance prediction of solar-only system. International Standard ISO 9459/2, 1995.
[6] Press WH, Teukolsky SA, Vetterling WT, Flannery BP. Numerical recipes in C. 2nd ed. Cambridge: Cambridge University Press; 1992, p. 656.