UNCERTAINTY ANALYSES IN SOLAR COLLECTOR MEASUREMENT

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Abstract – This paper presents a method for the evaluation of uncertainty of the parameters of an efficiency curve obtained by collector efficiency measurements. The method is in accordance with requirements that accredited laboratories have to fulfill. An example of actual measurement results is presented. This paper is also intended to serve as a basis for the development of a harmonized procedure for the evaluation of uncertainty in solar collector performance testing.

1. UNCERTAINTY – AN INDICATOR OF QUALITY

There are several reasons for a demand for a calculation of the uncertainty of solar collector testing: First of all, uncertainty values can provide a measure of reliability and accuracy of the testing result. Thus, uncertainty is a measure of the quality of a test and its results. Secondly, in laboratory intercomparisons, deviations of testing results among laboratories or among different testing methods can only be judged reasonably while considering the uncertainty or – related – the confidence interval of the result value(s) obtained by each laboratory. Thirdly, laboratories accredited according to EN 45001 or EN ISO 17025 are asked (in some countries even forced) to calculate the uncertainty of their results.

As uncertainty is a measure of quality, once established, uncertainty values might become a relevant factor for collector manufacturers when selecting an appropriate testing institute.

2. BASIC CONCEPT OF CALCULATION OF UNCERTAINTY

2.1 Definitions and terms

The concept of the calculation of uncertainty is described in the "Guide to the Expression of Uncertainty in Measurement" (GUM, 1995). However, although other examples are provided, no example of the case of a 3-parameter fit to measurement data is given.

The fundamental idea behind uncertainty is that the value of each measurand can only be determined with an uncertainty.

According to GUM (GUM, 1995), the concept of *uncertainty* involves the fact, that even if the effects of all *errors* have been corrected, the result of a measurement is still subject to an uncertainty of how well the result represents the value of the measured quantity.

Uncertainty values are expressed in the same way as "standard deviations". These values are often multiplied by a "coverage factor", resulting in an "expanded uncertainty" in order to obtain a value that encompasses a larger interval. The factor is typically in the range of 2 to 3. A factor of 2 corresponds to a level of confidence of approximately 95%.

Therefore VIM (VIM, 1993) defines the figure *uncertainty (of measurement)* as:
"Parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand."

2.2 Practical implementation of the calculation

The magnitude of the uncertainty is in practice influenced by a series of sources such as:

- The uncertainty of the calibration of the respective sensor, (influenced by each uncertainty in the chain of references that was used to calibrate the sensor),
- aging effects of the sensor since its last calibration,
- degree of dependence of each device to ambient conditions and the values of the ambient conditions of the respective ambient,
- the scatter of the measuring samples to be averaged to a "data point",
- the number of measuring points to be averaged,
- the magnitude of "engineer's guesses"
- others.

Therefore, all these sources have to be identified for each measurand in order to calculate the uncertainty of the final result. Problems that might occur and have occurred are:

- Calibration laboratories have to provide the information of the uncertainty of their calibration. Even if being accredited by a national accreditation body, some of the laboratories obviously have problems applying the procedure due to (at present) little demand,
- the keeper of the world standard for solar radiation (WRC in Davos/Switzerland) does not deliver an uncertainty for its standard, however this would be crucial for collector testing, Thus all connected calibration references (e.g. the one used by Kipp and Zonen, NL) can not calculate an uncertainty for their calibrations,

- specifications delivered in manuals (especially about drift and/or temperature dependence effects) are often too optimistic and some have been proven wrong.

2.3 Mathematical rules for calculation of uncertainty

Each uncertainty u is expressed as an uncertainty with a gaussian distribution. In this case, uncertainties can be added as root sum of squares

$$u = \sqrt{\sum_i u_i^{\ 2}} \quad (1)$$

The uncertainty caused by scatter of measurement samples within one data point is the "standard deviation of the mean" (GUM, 1995). It is calculated from the standard deviation σ of the samples divided by the root of the number of samples n:

$$u = \frac{\sigma}{\sqrt{n}} \quad (2)$$

In technical measurement we frequently find the situation (e.g. in sensor specifications) that the values are stated to lie between two bonds a_- and a_+, symmetrically arranged around the true value, the interval being $2a$. In this case the distribution is assumed to be equal between those bonds. In order to obtain a figure that fits into Eq. (1), the uncertainty of this distribution is (deduction: see GUM, 1995, Section C.3.2):

$$u = \frac{a}{\sqrt{3}} \quad (3)$$

If a measurand y depends on a number of (measured) quantities x_i to which uncertainties are associated, their contribution to the overall uncertainty of y is weighted by a sensitivity coefficient c_i, which is the partial derivative of y with respect to x_i.

$$c_i = \frac{\partial y}{\partial x_i} \quad (4)$$

And the uncertainty contribution $u_i(y)$ from x_i to y being

$$u_i(y) = c_i \cdot u(x_i) \quad (5)$$

However, not all uncertainties can be calculated in a strict mathematical manner. Some uncertainties have to be estimated from experience, e.g. the uncertainty of the area measurement of an absorber or the uncertainty of properties of a testing fluid other than water (although even the uncertainties of the properties of water might be worth a scientific discussion) or the drift of the sensor sensitivity with time since its last calibration.

3. CALCULATION OF UNCERTAINTY OF A 3-PARAMETER-FIT OF COLLECTOR DATA

In steady state solar collector testing the objective is, to calculate the uncertainty of the three resulting parameters from the uncertainties of the input values, which are: Measurements of area, temperatures, irradiance, flowrate as well as density (in case of volume flowrate measurement) and specific heat capacity of the testing fluid.

The evaluation of collector efficiency η using a 3-parameter-fit is expressed as (prEN 12975, 10/97)

$$\eta = \eta_0 + a_1 \cdot T_m^* + a_2 \cdot G T_m^{*2} \quad (6)$$

with

$$T_m^* = \frac{\Delta T_C}{G} \quad (7)$$

and

$$\Delta T_C = \left(\frac{T_{in} + T_{out}}{2} \right) - T_{amb} \quad (8)$$

From the input uncertainties for each of the M measurement points j the overall uncertainties of the three variables $T_m{}^*{}_j$, $GT^*{}_m{}^2{}_j$ and η_j are calculated ($GT_m{}^{*2}$ is treated as an independent variable). This is done by applying Eq. (5) to those variables with respect to every measurand and by summation according to Eq. (1).

At this point we see the chance of optimizing the fit according to a strategy that minimizes the resulting uncertainty. Thus, instead of minimizing simple least squares, we try to minimize a CHI-square function

$$\chi^2 = \sum_{j=1}^{M} \frac{(\eta_j - \eta(T_{mj}^*, GT_{mj}^*, \eta_0, a_1, a_2))^2}{u_j^{\ 2}} \quad (9)$$

where each element of the summation is weighted by the reciprocal value of the square of its uncertainty of the data point j with

$$u_j^{\ 2} = u(\eta)^2 + a_1^{\ 2} \cdot u(T_m^*)^2 + a_2^{\ 2} \cdot u(GT_m^{*2})^2 \quad (10)$$

This uncertainty depends on the fitted parameters, which means, that the minimization of Eq. (9) cannot be solved with a closed regression operation like a usual least squares problem, but has to be solved iteratively.

As is shown in Press et al., the uncertainties of the three parameters η, a_1, a_2 can be calculated from a matrix A, consisting of M lines and 3 columns. Each line consists of the values of the "basis functions" (the multiplying factors of the parameters to be identified, namely "1", " $T^*{}_m$" and " $GT^*{}_m{}^2$"), evaluated at the respective data point j, divided by their uncertainties according to Eq. (10):

$$A = \begin{pmatrix} \frac{1}{u_1} & \frac{(T_m^*)_1}{u_1} & \frac{(GT_m^{*2})_1}{u_1} \\ \frac{1}{u_2} & \frac{(T_m^*)_2}{u_2} & \frac{(GT_m^{*2})_2}{u_2} \\ \dots & \dots & \dots \\ \frac{1}{u_M} & \frac{(T_m^*)_M}{u_M} & \frac{(GT_m^{*2})_M}{u_M} \end{pmatrix} \qquad (11)$$

It can be shown (Press et al., 1986) that the operation

$$C = (A^T \cdot A)^{-1} \qquad (12)$$

yields a 3 x 3-matrix with the squared uncertainties of the three parameters as diagonal elements and the covariances between the parameters as off-diagonal elements.

Although Press et al. describe numerical methods for the calculation of the minimization of the CHI²-function and the matrix operations it should be pointed out that these operations can be carried out by basic features of commercially available spreadsheet programs.

Furthermore, the method is not limited to the measurement of a steady state collector efficiency curve as outlined above. It is in the same manner as well applicable to the “quasidynamic” test method described in prEN 12975-2 (10/97), however, the number of data points and the number of parameters is much higher.

4. EXAMPLE RESULT

The method described above has been applied to several collectors tested at SPF in Rapperswil, Switzerland. One of the results is presented here:

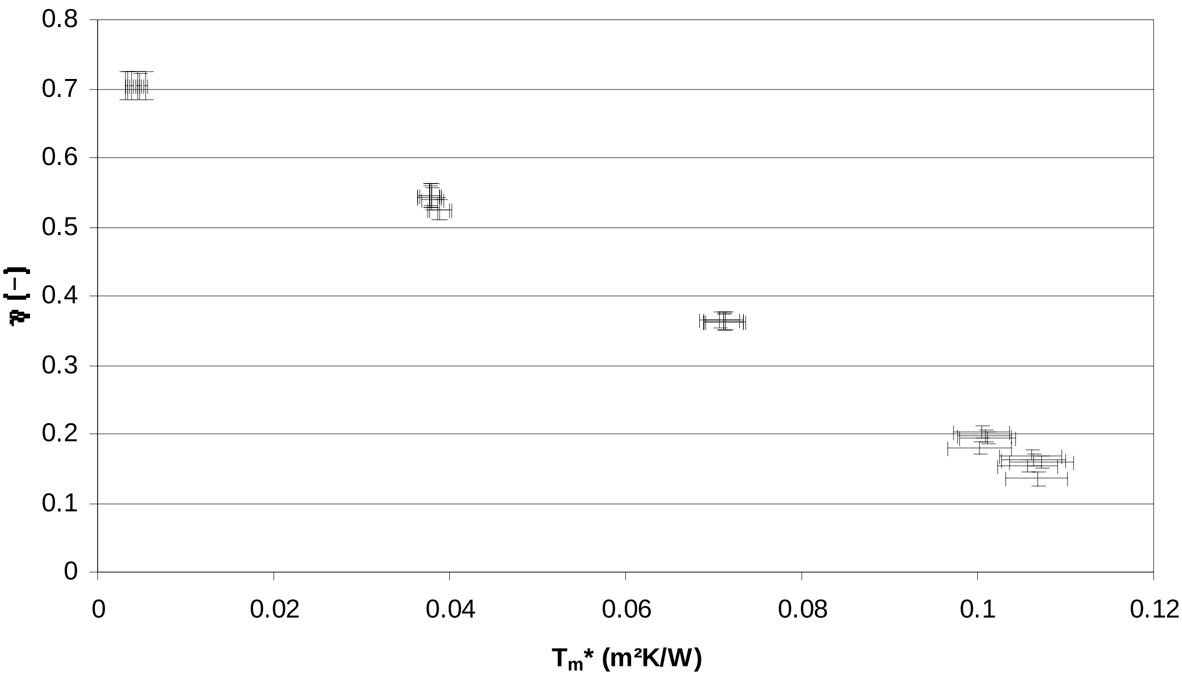

Fig. 1: Data points and uncertainty bars, coverage factor = 2

As can be seen from the diagram, the ranges of uncertainty look reasonable, but the uncertainties in T^*_m increase with T^*_m. Note that the uncertainties in $GT^*_m{}^2$ cannot be visualized in this diagram.

The evaluated parameters together with their expanded (coverage factor = 2, corresponding to a level of confindence of approximately 95%) uncertainties are:

	η_0 (-)	a_1 (W/m²K)	a_2 (W/m²K²)
estimate	0.724	-4.6292	-0.0075
uncertainty	0.010	0.0006	0.005

Table 1: Parameter values

The result can be presented as an efficiency curve and two delimiting curves representing the values of the efficiency plus (minus) the expanded uncertainty.

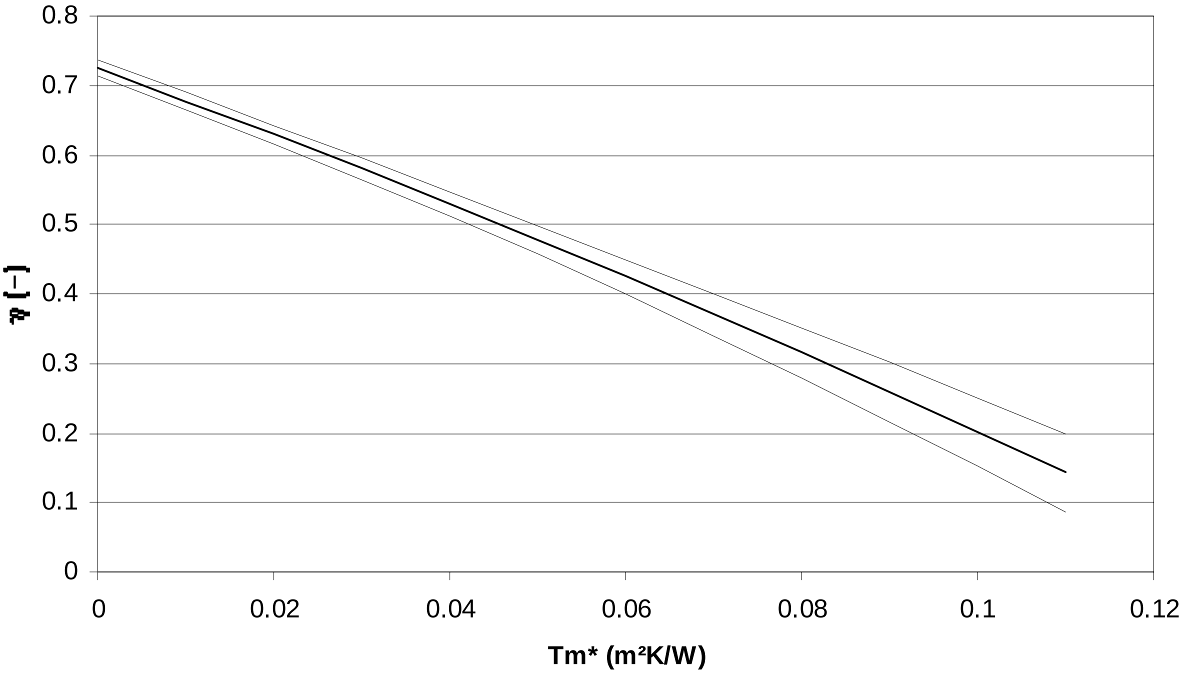

Fig. 2: Efficiency curve at G=800 W/m² and uncertainty range, coverage factor = 2

The values and the graphs presented above include a "guess" uncertainty of the irradiance measurement of 0.6% assumed to be caused by drifts observed between two pyranometer calibrations.

If the testing engineer "forgets" (by way of example) this source of uncertainty, we obtain the following values:

	η_0 (-)	a_1 (W/m²K)	a_2 (W/m²K²)
estimate	0.724	-4.6280	-0.0074
uncertainty	0.009	0.0004	0.003

Table 2: Parameter values, less uncertainty in irradiance

The values found for the parameters are more or less the same (not necessarily: Eq. (9) is affected!), but the uncertainties associated with the parameters have decreased significantly.

It should be added that prEN and ISO standards state *accuracy* bonds for the relevant measurands. Although the terms used in those standards are not compatible with the terms in the field of *uncertainty* defined in VIM and GUM, the uncertainties that are "acceptable" according to the testing standards, are substantially larger than those reached by careful calibration of sensors of high quality.

5. DISCUSSION OF METHOD AND RESULTS

The method presented here is (as far as the method is concerned) clear, straightforward and repeatable. However, as can be seen from the examples, the size of the uncertainties is extremely upon the assumptions made by the testing laboratory regarding their measurements and fluid.

The result is not only affected by evaluating the size of commonly known sources of uncertainties. Also considering and finding sources of uncertainties that are specific of measurements of solar collectors affects the result.

The following problem arises: The more intensively the testing institute explores possible sources of uncertainties and the more effort the engineers undertake, the more they "worsen" the quality figures of their own testing results by raising the uncertainty values.

Moreover, there are testing institutes in Europe who state no or only the mere standard deviation of the parameters of the simple least squares fit together with the result. This is no statement about the "uncertainty" as defined in VIM and GUM, but only a measure of the scatter of the measuring points.

As uncertainty and standard deviations figures look "similar" for the reader of a test report (or even seem to be "zero" when they are not indicated at all), there is a considerable risk of confusion. It must be pointed out that the two methods (standard deviation of least squares fit and uncertainty calculation) are different, their results have a different meaning, and, above all, the method described in this paper involves far greater efforts spent on the work and the maintenance of the testing equipment and is thus far more expensive.

6. CONCLUSIONS

- Solar collector testing is affected by uncertainty like any other measurement task.
- The methods for evaluating this uncertainty are well documented in the literature, namely in GUM and Press et al., however, regarding solar collectors, both parties, testers and readers of test reports, are not yet familiar with the method.
- The more effort the testing institute undertakes in identifying uncertainties, the larger the values and the "poorer" is the quantifiable quality of the test result.
- "Forgetting" sources of uncertainty "improves" the uncertainty figures.
- There are calibration institutes who have not introduced uncertainty calculation procedures. This undermines the efforts of the authors of GUM and EN 45001 (EN ISO 17025.
- We therefore suggest that solar collector testing institutes develop and agree on a common (maybe even standardized) procedure for calculation of uncertainty, including a definition of the sources of uncertainty to be evaluated and, in case of absence of better knowledge, on assumptive values.
- The proposed agreement should also include a policy of how to present the uncertainty values in test reports and how to clearly differentiate "real" uncertainty values from values obtained from other methods.

REFERENCES

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Guide to the Expression of Uncertainty in Measurement (1995) (GUM, 1995), ISO, Switzerland.

Press W. H. et al. (1986) *Numerical Recipes,* Ch. 14.3. Cambridge University Press, Cambridge.

FOR FURTHER READING

Mathioulakis E. et al. (1999). Assessment of Uncertainty in Solar Collector Modeling and Testing Solar Energy Vol. 66. No. 5 pp. 337-347. Elsevier Science.

Expression of the Uncertainty of Measurement in Calibration (1999), European co-operation for Accreditation (document "EA-4/02") (due to small inconsistencies, never read without reading GUM)