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# ASSESSMENT OF UNCERTAINTY IN SOLAR COLLECTOR MODELING AND TESTING

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**Abstract**—The basic scope of solar collector testing is the determination of the collector efficiency by conducting measurements under specific conditions defined by international standards. The experimental results of testing lead to determination of the parameters of a more or less complex model, usually a 2- or 3-parameter single node steady-state model, which describes the collector behavior. In the present study, a systematic analysis of the contribution of all the uncertainty components on the basis of the ISO 9806-1 test procedure is carried out in order to determine the final uncertainty in the characteristic equation parameters and the instantaneous efficiency of the collector. A step-by-step methodology, based on specific statistical tools, for evaluation of the suitability of the collector models already in use, is proposed. This methodology not only allows an evaluation of the reliability of the testing procedure itself, but also a quantification of the goodness-of-fit. Furthermore, if the uncertainty in the characteristic equation parameters is known, the uncertainty in the collector instantaneous efficiency to be predicted can be assessed. This is essential for the reliability of the results of design tools, for which collector efficiency is a key parameter. 

## 1. INTRODUCTION

The behavior of solar thermal systems regarding energy output is an operational factor of great importance for systems utilizing solar energy. This factor affects and influences the design of solar thermal installations and the economic data concerning their exploitation.

To determine collector efficiency, a standard and commonly accepted test procedure is necessary. This procedure should be accurate enough to permit a meaningful characterisation and ranking of candidate collectors.

In the procedure for collector evaluation, the key point is the determination of the parameters of a model capable of satisfactorily describing the energy behavior of the collector. The equation derived is considered to express the specific collector and can subsequently be used to predict its output under any conditions.

In the present study, the ISO Standard 9806-1 (Test Methods for Solar Collectors. Part 1: Thermal Performance of Liquid Heating Collectors Including Pressure Drop) is examined, mainly due to its extensive use and international application (ISO, 1994). At this point it has to be noted that the philosophy of ISO 9806-1 is very close to that of the various national or European standards (ASHRAE, 1978; AFNOR, 1980) and that the methodology which will be presented here could be applied to these standards as well, without major changes.

The basic target of solar collector testing is the determination of the collector efficiency by measurements under specific conditions (Daffie and Beckman, 1991). More specifically, it is assumed that the behavior of the collector can be described by a 2- or 3-parameter single node steady-state model $n = f(T_i^*)$:

$$n = n_0 - U_0 T_i^* \tag{1a}$$

$$n = n_0 - U_1 T_i^* - U_2 G (T_i^*)^2 \tag{1b}$$

The above equations (1a) and (1b) as well as the whole analysis presented in this paper are also valid for reduced temperature difference $T_m^*$ calculated with respect to the mean collector fluid temperature. In this case the variable $T_i^*$, where it appears, must be replaced by $T_m^*$.

It is noted that several more elaborated models and testing methods have been proposed by various authors (Perers, 1997). However, in this

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paper, the testing method defined in Standard ISO 9806-1 is chosen, since this method is the only one acceptable and standardized in an international level to date. Furthermore, by considering this method, the calculation of test results uncertainty is of great practical importance.

During the experimental phase, the output, solar energy and the basic climatic quantities are measured. In analyzing the data, a least-squares fit is performed on the measured data, in order to the determine parameters $n_0$ and $U_0$ or $n_0$, $U_1$ and $U_2$.

In practice, this procedure determines only the equation of the collector behavior without calculating the uncertainties in the determined parameters, and thus the suitability of the concerned model is not evaluated with statistical criteria. Despite the widespread use of testing and the great importance of the testing results, an objective and standardized method for the determination of uncertainty in test results is still lacking. The question of uncertainty is crucial if one wishes:

1. To examine the quality of each model, i.e. its ability to describe collector behavior, as this was found experimentally, and to compare the efficiency of each model against well-known test statistics. This can be done by the use of $\chi^2$, or $\chi^2$ merit function and the goodness-of-fit quantified by the probability $Q$ of the data not fitting the model by chance.
2. To determine the parameters of the collector characteristic equation, also considering the experimental uncertainties, using a Weighted Least Squares (WLS) instead of the simple Least Squares (LS).
3. To identify the uncertainty in the parameters and to decide as to their validity with specific statistical criteria. These criteria concern the ratio of the uncertainty in each parameter to its value (it must be less than unity) and their covariance (which should be very small as an indication of their independence).

It is noted here that only a limited number of publications deal with the accuracy of test results of solar thermal devices. A comprehensive treatment of the accuracy of test procedures of solar water heaters is provided by Bourges *et al.* (1991a,b). A corresponding analysis for solar collector testing methods has been proposed by Proctor (1984a,b,c). In this analysis only the uncertainties related to measuring device errors and the standard least-squares technique have been considered. However, as will be discussed later on, this approach is equivalent to assuming a good fit and prohibits an independent assessment of goodness-of-fit.

In this publication we develop, step by step, the general rules of uncertainty analysis and their application in a typical case of a commercial collector tested according to ISO 9806-1. The test results were obtained by the Solar and other Energy System Lab which operates under the EN45001 Quality Assurance System, with strict adherence to the requirements of the testing standard.

## 2. THE TESTING METHOD IN BRIEF

Testing according to ISO 9806-1 concerns measurement of the collector efficiency under steady-state conditions, in specific operation conditions. It is performed according to the following procedure:

1. The collector is placed on a stand, which can be moved in such a way that the solar irradiance incident on the collector plane can be vertical.
2. Water flows through the collector at a constant flowrate during all the measurements. The average surrounding air speed has to be in the range from 2 m s$^{-1}$ to 4 m s$^{-1}$ during all measurements.
3. A temperature for the water in the collector inlet is selected (set-point). This is kept constant during measurements concerning this set-point.
4. Assuming that the whole procedure is under steady-state conditions, the following quantities are measured: global solar irradiance $G$, ambient air temperature $T_a$, water flowrate $m$, and temperatures in the collector inlet, $T_{in}$, and outlet, $T_{out}$, respectively.
5. Measurements for each state set-point derive from the average values of the measurements over a 15 min period, during which the deviations of the values of $T_{in}$, $m$ and $G$ must be lower than the specific limits.
6. Steps (1)–(5) are repeated until at least 16 points, four points for each temperature set-point (collector inlet) are taken. The set-points are selected so that they cover the whole range of the collector operation.

After this, a typical correlation problem is solved, from which the parameters of Eqs. (1a) and (1b) are determined from the experimental data.

The standard is confined to determine the accuracy of measurements and does not provide any kind of treatment of experimental data for the determination of uncertainty.

## 3. UNCERTAINTIES ASSOCIATED WITH EXPERIMENTAL DATA

### *3.1. General principles for the determination of uncertainty*

Standard uncertainties in experimental data are determined by taking into account Type A and Type B uncertainties. According to the recommendation of ISO VIM (1995), the former are the uncertainties determined by statistical means while the latter are determined by other means. At this stage, it is important to clarify the differences between the several kinds of uncertainties by taking into account their source and the way in which they are determined (ISO, 1995; ISO VIM, 1995).

A characteristic example is the measurement of ambient air temperature $T_a$. The standard specifies that the sensor has to be placed in a specific position with respect to the collector and that a measuring device characterized by 'an accuracy of $\pm 0.50°C$' must be used. The uncertainty which is associated with the value of $T_a$ for each measurement point, i.e. the difference that can exist between the final value and the true average value of the ambient air temperature around the collector, is the result of:

1. the uncertainty of the measuring instrument (Type B uncertainty), which is a characteristic feature of the instrument itself.
2. the uncertainty which is determined statistically (Type A uncertainty), which represents the deviation of the measured value during sampling of data that will be used for the determination of one point; and
3. the uncertainty which derives from the fact that the temperature at the point of measurement at which the sensor is placed may not represent the true air temperature in the surrounding of the collector.

Uncertainties (1) and (2) can and must be determined from the data of the measuring instrument and the specific measurements. On the contrary, uncertainties (3) cannot be determined quantitatively, and are a function of the quality of the testing method. The influence of this last category of uncertainties will be incorporated into the final result and will affect the ability of the specific model to describe the collector behavior.

Generally, in cases where an attempt is made to describe the behavior of a certain system with an approximate model, a distinction on the following lines should be made:

- On the one hand, the uncertainties which characterize every measurement itself and which are related to the quality of the measuring instrument and the stability of the measurement. These uncertainties can be determined quantitatively.
- On the other hand, the uncertainties which are related to the degree to which the measurement or the model is representative, and which characterize the quality of the methodology followed. These uncertainties cannot be determined quantitatively and, after all, their determination has no meaning. Their influence is reflected in the ability of the methodology used (model and testing method) to describe the phenomenon. If, for example, it is proved that certain experimental results are not represented satisfactorily by Eq. (1a) or (1b), the whole methodology or the suitability of the specific equation is in question.

### *3.2. Type A uncertainties*

In our case, Type A uncertainties derive from the statistical analysis of the repeated measurements at each point of the steady-state operation of the collector. It should be brought in mind that, according to the standard, $N$ measurements are taken for 15 min (about 30 measurements), and the average value for each measured quantity is found. For every operation point of the collector, the best estimate of a quantity $X$ is the arithmetic mean $\bar{x}$ of the $N$ observations $x_j$ and its Type A uncertainty is the *standard deviation of the mean* (Fuller, 1987):

$$\sigma_{A,X} = \left( \frac{\sum_{j=1}^{N} (x_j - \bar{x})^2}{N(N-1)} \right)^{0.5} \tag{2}$$

By nature, Type A uncertainties depend on the specific conditions of the test. Thus, they include the fluctuations in the measured quantities during the test which lie within the limits imposed by the standard, and also the fluctuations in the testing conditions not considered by the model. Such fluctuations concern, for example, the air speed or the percentage of global diffuse irradiance.

### *3.3. Type B uncertainties*

Type B uncertainties derive from the calculation of uncertainties over the whole measurement, taking into account all available data, such as sensor uncertainty, data logger uncertainty etc. The standard defines the upper limits of the accuracy of the measurements. This accuracy must not be worse than the values given in Table 1.

It is important to make a remark concerning the use of term *accuracy*. According to the guidance of ISO (1995), the term 'accuracy' is a qualitative determination and represents the closeness of the result of a measurement to the true value of the measured quantity. In view of this, the expression *uncertainty* would be preferable if one wishes to quantify the accuracy.

One can obtain the standard uncertainty $u_{B,x}$ associated with the required accuracy $\alpha_x$ for a Type B evaluation by assuming that the stated accuracy provides symmetric bounds to an additive correction of expectation equal to zero, with an equal probability of lying anywhere within the bounds. In this case, the standard Type B uncertainty in the estimate $x$ of the measurand $X$ can be obtained using the following equation (ISO, 1995; Dietrich, 1991):

$$u_{B,x} = \left(\frac{\alpha_x^2}{3}\right)^{0.5} \quad (3)$$

If there are more than one independent sources of uncertainty, (Type B or Type A) $u_i$, the final uncertainty is calculated according to the general law of uncertainties combination (Dietrich, 1991):

$$u = \left(\sum_i u_i^2\right)^{0.5} \quad (4)$$

The uncertainty in the measurements of the pyranometer is found by taking into account two basic sources of error: non-linearity error and errors associated with the temperature dependence of sensitivity. The remaining errors (spectral sensitivity, cosine and azimuth response and response time) are considered negligible, in view of the fact that measurements are performed under clear sky with vertical incidence and practically constant irradiance. In the case of the secondary standard pyranometer used for the measurements for this study, the respective accuracies are as follows (Kipp & Zonen, 1992):

non-linearity error (for the range 500–1000 W m$^{-2}$): $\alpha_{g1} = \pm 5$ W m$^{-2}$;
- temperature dependence: $\alpha_{g2} = \pm 5$ W m$^{-2}$ within the actual operating range.

Consequently, by applying the law of propagation of errors, the respective standard uncertainties are given by the following expression:

$$u_B(g) = \left(\frac{(a_{g1})^2}{3} + \frac{(a_{g2})^2}{3}\right)^{0.5} = 4\ \text{W m}^{-2} \quad (5)$$

Generally, the application of Eq. (3) to the calculation of Type B uncertainty leads to the values of Table 2.

### 3.4. Combined uncertainty

The term *combined standard uncertainty* means the standard uncertainty in a result, when that result is obtained from the values of a number of other quantities. In most cases a measurand $Y$ is determined indirectly from $N$ other quantities $X_1$, $X_2, \ldots, X_N$ through a functional relationship $Y = f(X_1, X_2, \ldots, X_N)$. The standard uncertainty in the estimate $y$ is given by the *law of error propagation* (ISO, 1995; Fuller, 1987):

$$u_y = \left(\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j)\right)^{0.5} \quad (6)$$

where $u(x_i, x_j)$ is the covariance associated with $x_i$ and $x_j$. For the case where all of the input estimates are not correlated with $u(x_i,x_j) = 0$, Eq. (6) reduces to Eq. (7):

$$u_y = \left(\sum_{i=1}^{N}(\frac{\partial f}{\partial x_i})^2 u_{x_i}^2\right)^{0.5} \quad (7)$$

In our case, Eq. (6) is used for the evaluation of combined uncertainty in the efficiency values $n$ and of the reduced temperature difference $T_i^*$,

Table 1. Required accuracy of measurements according to ISO 9806-1

| Quantity | Accuracy |
| --- | --- |
| Water temperature | $\pm 0.1$°C |
| Temperature difference $\Delta T$ | $\pm 0.1$°C |
| Ambient air temperature $T_a$ | $\pm 0.5$°C |
| Flowrate | $\pm 1.0$% (of reading) |
| Collector aperture | $\pm 0.1$% (of reading) |
| Solar irradiance | Pyranometer Class 1 (according to ISO 9060) |

Table 2. Type B uncertainties in measurements

| Quantity | Type B uncertainty |
| --- | --- |
| Water temperature | $u_{B,T_{in}} = u_{B,T_{out}} = 0.06$°C |
| Temperature difference $\Delta T$ | $u_{B,\Delta T} = 0.06$°C |
| Ambient air temperature $T\alpha$ | $u_{B,T_a} = 0.29$°C |
| Flowrate | $u_{B,m} = 0.0058$ m |
| Collector aperture | $u_{B,A_c} = 5.8 \times 10^{-4} A_c$ |
| Solar irradiance | $u_{B,G} = 4$ W m$^{-2}$ |

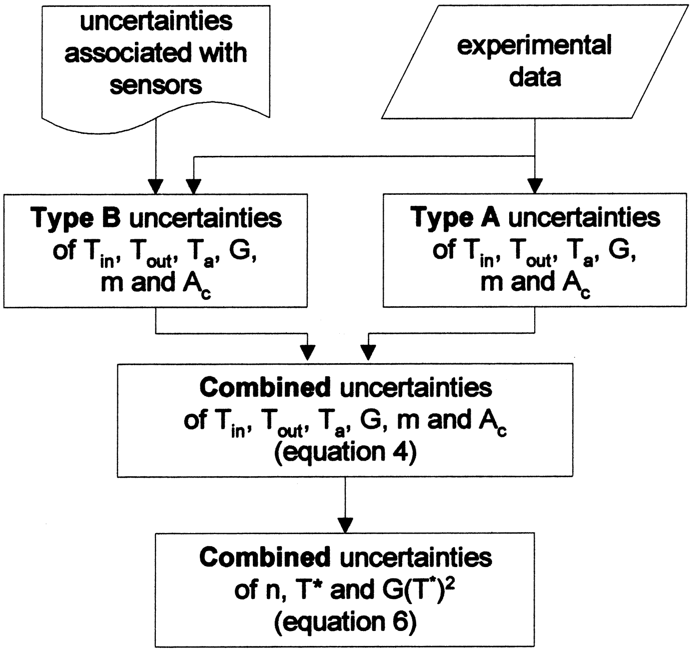


Fig. 1. Propagation of uncertainties.

which are calculated as a function of $T_{in}$, $T_{out}$, $\Delta T$, $T_a$, $m$, $G$ and $A_c$. The calculation is conducted following the steps described in the flow chart of Fig. 1.

Fig. 2 shows the expanded standard uncertainties $\sigma_n$ and $\sigma_{T*}$ of $n$ and $T_i^*$, respectively, as calculated for a specific collector for each measurement point. The horizontal bars refer to $\sigma_{T*}$ and the vertical ones to $\sigma_n$. In order to show the figure more clearly, only some of the 32 points (measured in the laboratory) are presented. All measurements satisfy the requirements of the standard, using calibrated measuring instruments. Despite the fact that these uncertainties concern the specific collector, the results of other collectors are in the same range, as was proved after

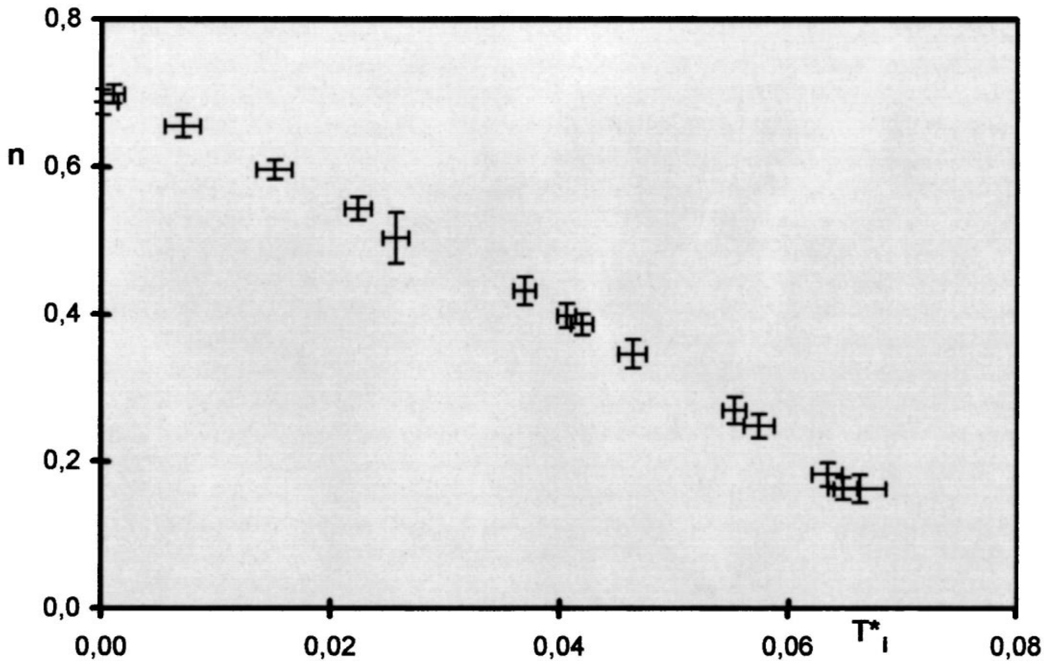


Fig. 2. Values of $n$, $T_i^*$ and combined standard uncertainties in $n$, $T_i^*$.

many tests in the laboratory. This happens because Type A uncertainties are related to the stability limits of the testing conditions mentioned in the standard, whereas Type B uncertainties are derived from the required accuracy.

It is also noted that the values of the uncertainties are presented in Fig. 2 as expanded uncertainty $\sigma_x$, as is the usual practice. The expanded uncertainty in an estimate $x$ is obtained by multiplying the combined standard uncertainty $u_x$ by a coverage factor $k=2$, corresponding to a level of confidence of 95% (ISO VIM, 1995). Although the exact way in which expanded uncertainty is calculated goes beyond the objective of this study, it is worth noting briefly that the meaning of the above is that the probability of the true value of $X$ lying within the range $[x-\sigma_x, x+\sigma_x]$ is 95%.

In the case of the 3-parameter model the quantity $G(T_i^*)^2$ is treated as an independent variable, thus its uncertainty $\sigma_{G(T*)^2}$ is calculated separately, by applying the law of propagation of errors on equation $G(T^*_i)^2=(T_i-T_a)^2/G$. In fact a 2-dimensional linear fit is required, since a single variable $n$ is modelled as a function of two variables $T_i^*$ and $G(T_i^*)^2$.

## 4. FITTING

### *4.1. Least squares and weighted least squares in general*

The general problem of fitting is to find a model with $M$ parameters $a_j$ to represent a series of $N$ observations $(x_i, y_i)$ with the greatest accuracy:

$$y(x)=y(x; a_1 \ldots a_M) \tag{8}$$

In the above equation a single variable $y$ can be a function of either a single variable $x$ or a vector $x$ of more than one variable, in the case of a multidimensional model. The basic methodology is always the same (Press *et al.*, 1996; Dietrich, 1991): a *figure-of-merit function* is selected, to give an indication of the difference between the real data and the model. After this, the model parameters are selected so that the value of the function is minimized. The deviations of the model from the real data can be attributed to experimental errors but also to model weaknesses.

The least squares (LS) method tries to give an answer to this question: *given a set of parameters $a_1, a_2, \ldots, a_M$, what is the probability that this set is the desired one?* Assuming that every point of our data is associated with an error which follows a normal distribution around the 'true' value with standard deviation $\sigma$ which is the same for all points, the maximalization of the probability that this is the correct set of parameters leads to the minimization of the function

$$\sum_{i=1}^{N}[y_i-y(x_i; a_1, a_2, \ldots, a_M)]^2 \tag{9}$$

The problem with this approach is that, in reality, the typical deviation $\sigma$ is almost never constant and the same for all points, but that each data point $(x_i, y_i)$ has its own standard deviation $\sigma_i$. Another very interesting alternative is the use of the *weighted least squares (WLS)* method, which calculates, on the basis of the measured values and their uncertainties, not only the model parameters but also their uncertainty. In this way a qualitative evaluation of fitting can be performed.

In the case of WLS, the maximum likelihood estimate of the model parameters is obtained by minimising the $\chi^2$ function (Press *et al.*, 1996):

$$\chi^2=\sum_{i=1}^{N}\frac{(y_i-y(x_i; a_1, a_2, \ldots, a_M))^2}{u_i^2} \tag{10}$$

where $u_i^2$ is the variance of the difference $y_i-y(x_i; a_1, a_2, \ldots, a_N)$:

$$u_i^2=\mathrm{Var}(y_i-y(x_i; a_1, a_2, \ldots, a_M)) \tag{11}$$

Since the parameters $a_1,..., a_M$ are to be calculated, not all the terms that appear in Eqs. (9) and (10) are statistically independent, for this the degrees of freedom are $\nu=N-M$.

It emerges from Eq. (11) that the quantity $u_i^2$ depends on the experimental uncertainties $u_{x_i}$ and $u_{y_i}$. With this consideration in mind, the $\chi^2$ merit function actually gives an idea about the relation between the model deviation from the experimental data and the uncertainties in the measurements. A relatively good model will be able to explain the deviations observed on the basis of the experimental errors and the corresponding $\chi^2$ function will have a value close to $\nu$. Among the advantages of the use of the weighted least squares is the fact that the real experimental uncertainties are taken into account in determining the model parameters, the fact that it allows the calculation of the uncertainties in these parameters, and also that it gives a realistic estimation of goodness-of-fit. However, even in the case that a least-squares fitting is selected by neglecting the uncertainties $u_i$ in the phase of the calculation of parameters $a_1$, ..., $a_M$, the $\chi^2$ function and the goodness-of-fit can still be determined afterwards using Eq. (10).

From the values of $\chi^2$ and $\nu$ the probability $Q(0.5\nu, 0.5\chi^2)$ *that the data do not fit the model by chance* can be calculated (Press *et al.*, 1996, Bajpai *et al.*, 1977):

$$Q(a,x) = \frac{1}{\Gamma(a)}\int_x^\infty e^{-t}t^{a-1}\,dt; \quad \text{with } \alpha > 0, \text{ and } \Gamma(a) = \int_0^\infty t^{a-1}e^{-t}\,dt \tag{12}$$

The probability $Q$ can be explained as a quantitative indication of goodness-of-fit for the specific model. Generally speaking, if $Q$ is larger than 0.1, then the goodness-of-fit is believable. If it is larger than 0.001, then the fit may be acceptable, under certain conditions. If $Q$ is less than 0.001, then the model (or the estimation procedure) can be called into question.

In the case of solar collectors, where a 2- or 3-parameter model is concerned, by applying Eqs. (7) and (11), the denominator in Eq. (10) is written as follows:

$$\text{2-parameter model, } Y = a + bX: \quad u_i^2 = u_{y_i}^2 + b^2u_{x_i}^2 \tag{13a}$$

$$\text{3-parameter model, } Y = a + bX1 + cX2: \quad u_i^2 = u_{y_i}^2 + b^2u_{x1_i}^2 + c^2u_{x2_i}^2 \tag{13b}$$

So, the purpose is to minimize Eq. (10) with respect to $a_1, \ldots, a_M$. Unfortunately, as can be seen from Eqs. (10), (13a) and (13b), the occurrence of $b$ and $c$ in the denominator makes the Eq. (10) non-linear. Its solution by analytical methods is possible only if the uncertainty in $x_i$ can be considered negligible (Press *et al.*, 1996). Otherwise, the solution is possible by using numerical methods for minimisation of non-linear functions.

Generally, the requirements for the acceptance of a good fitting can be reported as follows (ISO, 1995; Press *et al.*, 1996, Bajpai *et al.*, 1977):

1. The goodness-of-fit, i.e. the probability $Q(0.5\nu, 0.5\chi^2)$ that the data do not fit the model by chance, should be high or, equivalently, the $\chi^2$ statistic should be about the number of degrees of freedom.
2. The determined parameters $a_1, \ldots, a_M$ should be independent, i.e. $\mathrm{Covar}(a_i, a_j) \ll 1$.

### *4.2. Fitting in solar collectors testing*

Standard ISO 9806-1 is limited to the definition of the method for the determination of the model parameters, specifically the least-squares method. This procedure is equivalent to assuming a good fit and prohibits an independent assessment of goodness-of-fit. This has two basic consequences: firstly, no kind of quality control of the fitting can be made; and, secondly, the determination of the uncertainty of the parameters is not possible. Furthermore, an indirect consequence of the above is that the effective comparison of test results between different test laboratories can not be done. Generally, the test results cannot be characterized qualitatively, leading to a lowering of their reliability when they are to be used, for example, in simulation of solar hot water systems.

As explained in the previous sections, these deficiencies can be overcome by employing the weighted least squares. The fact that the experimental data are subject to measurement errors in both coordinates leads to the use of Eqs. (10)–(13b). The whole procedure is depicted in the chart of Fig. 3.

The first step is to determine the values $n_0$ and $U_0$ for a 2-parameter model (or $n_0$, $U_1$ and $U_2$ for a 3-parameter model) for which the function $\chi^2$ is minimized. For the minimisation the Levenberg-Marquardt method for non-linear parameter estimation is used, and attention is given to ensure that local minimums are avoided. The search for the minimum can become faster if the initial values of the parameters are the ones calculated analytically by unweighted least squares.

Then, $\chi^2$ and the goodness-of-fit are calculated from Eqs. (10) and (12). It has to be noted here that the calculation of $\chi^2$ and the goodness-of-fit is possible even in the case of unweighted least squares using the parameter values as these are calculated from the respective analytical equations.

Finding the standard uncertainties $u_{n_0}$, $u_{U_0}$, $u_{U_1}$ and $u_{U_2}$ in parameters $n_0$, $U_0$, $U_1$ and $U_2$ is more complicated, because of the non-linearity present in Eq. (10). Our strategy is therefore to find these uncertainties numerically. The method for the case of a 3-parameter model is presented below; for a 2-parameter model the same methodology is followed. For a more detailed review of the mathematics of the method, see Press *et al.* (1996).

Let $K$ be a matrix whose $N \times M$ components $k_{i,j}$ are constructed from $M$ basic functions evaluated at the $N$ experimental values of $T_i^*$ and $G_i(T_i^*)^2$

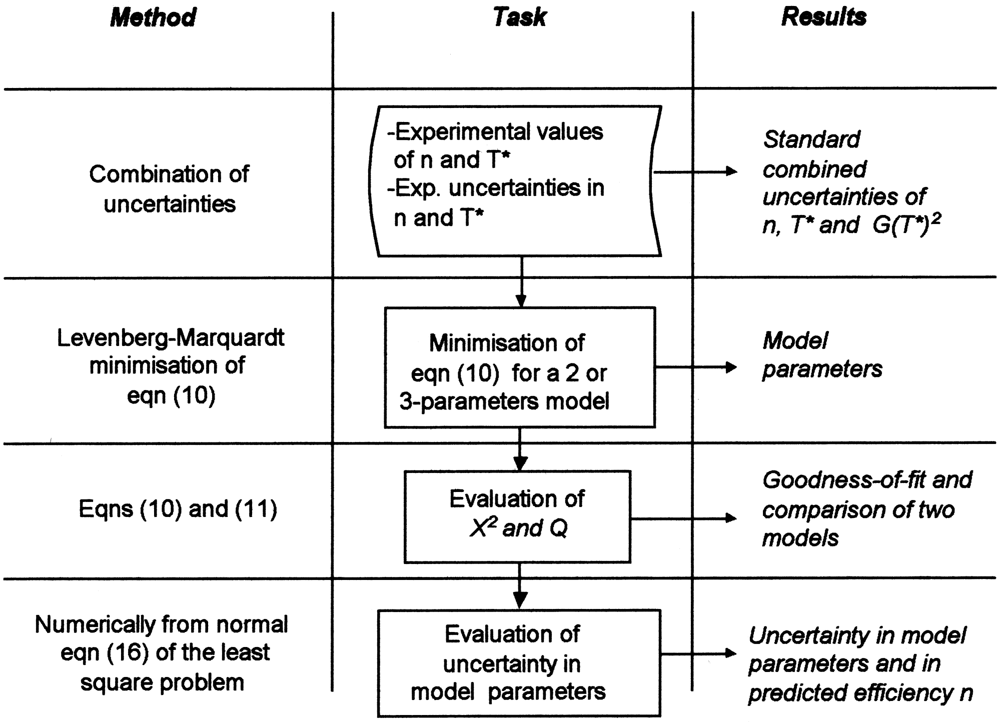


Fig. 3. Synopsis of fitting procedure.

weighted by the uncertainty $u_i$ ($M=2$ or $M=3$ for a 2- or 3-parameter model, respectively):

$$k_{i,1}=1/u_i,\ k_{i,2}=T_i^*/u_i,\ k_{i,3}=G(T_i^*)^2/u_i,$$

$$K=\begin{vmatrix} k_{1,1} & \cdot & \cdot & k_{1,M} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ k_{N,1} & \cdot & \cdot & k_{N,M} \end{vmatrix} \tag{14}$$

Let also $L$ be a vector of length $N$ whose components $l_i$ are constructed from values to be fitted, weighted by the uncertainty $u_i$:

$$l_{i,j}=n_i/u_i,\quad L=\begin{vmatrix} n_1/u_1 \\ \cdot \\ \cdot \\ \cdot \\ n_N/u_N \end{vmatrix} \tag{15}$$

The *normal equation of the least-squares problem* can be written:

$$(K^{\mathrm{T}}\cdot K)\cdot \mathrm{INV}(C)=K^{\mathrm{T}}L \tag{16}$$

where $C$ is a matrix whose diagonal elements $c_{i,i}$ are the variances (squared uncertainties) of the fitted parameters and the off-diagonal elements $c_{i,j}$, $i\neq j$, are the covariances between these parameters. Eq. (16) can be solved by a standard method, for example, by Gauss-Jordan elimination. It should be noted that the calculation of covariances between the fitted parameters is necessary to estimate the acceptance criteria of fitting and to calculate the uncertainty in the predicted values of efficiency $n$ given.

The calculation of collector efficiency for given values of irradiance and inlet water temperature can be easily done by entering the calculated parameters in Eqs. (1a) and (1b). The uncertainty in the predicted values of $n$ is calculated by Eqs. (17) and (18) for the 2- or 3-parameters model, respectively. Eqs. (17) and (18) derive from Eq. (6), where the values of irradiance and inlet water temperature are supposed to be known without uncertainty. Similar relations can also be derived from Eq. (6) in the case that the values of irradiance and inlet water temperature are accompanied by known uncertainties.

$$u_n=\left[u_{n_0}^2+(T_i^*)^2u_{U_0}^2+2T_i^*\mathrm{Cov}(n_0,U_0)\right]^{0.5} \tag{17}$$

$$u_n=\Big[u_{n_0}^2+(T_i^*u_{U_1})^2+\left(GT_i^{*2}u_{U_2}\right)^2 + 2T_i^*\mathrm{Cov}(n_0,U_1)+2GT_i^{*3}\mathrm{Cov}(U_1,U_2) + 2GT_i^{*2}\mathrm{Cov}(n_0,U_2)\Big]^{0.5} \tag{18}$$

### 4.3. Results

The results presented here concern the typical collector, mentioned in a previous stage. It should

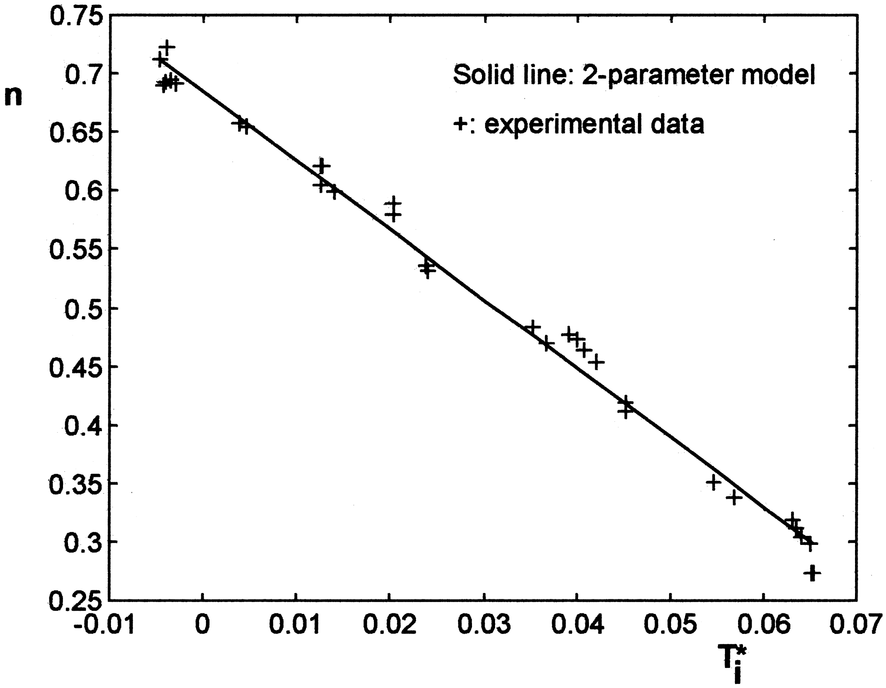


Fig. 4. Experimental data and 2-parameter model (WLS fitting).

be stressed that the efficiency and the reduced temperature difference $T_i^*$ were calculated for 32 steady-state operation points, as well their respective uncertainties (see Section 3.4).

The experimental data, together with the 2-parameter WLS fitting, are shown in Fig. 4, while Fig. 5 shows the respective 3-parameter WLS fitting. The graphical representation of the 3-

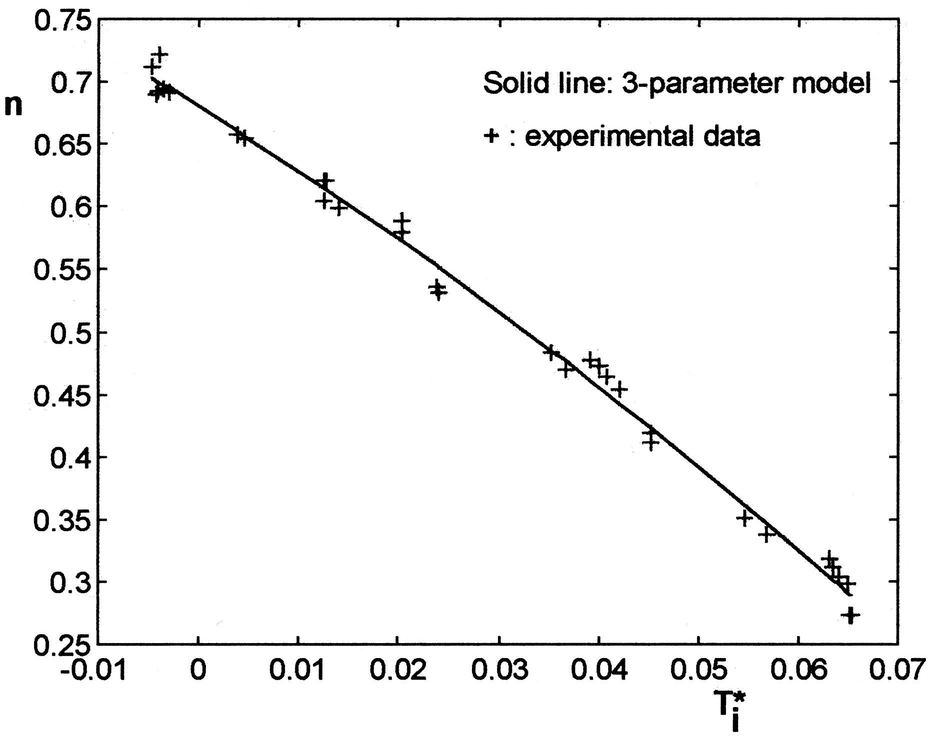


Fig. 5. Experimental data and 3-parameter model (WLS fitting).

Table 3. Results obtained using Least Squares (LS) and Weighted Least Squares (WLS) for a 2-parameter model

| Model: $n=n_0-U_0T_i^*$ | | |
|---|---|---|
| Quantity | Method of fitting | |
| | LS | WLS |
| $n_0$ | 0.6829 | 0.6848 |
| $U_0$ | 7.830 $K^{-1}$ W $m^{-2}$ | 5.855 $K^{-1}$ W $m^{-2}$ |
| $\sigma_{n_0}$ | – | 0.0450 |
| $\sigma_{U_0}$ | – | 0.1183 $K^{-1}$ W $m^{-2}$ |
| $\mathrm{Cov}(n_0, U_0)$ | – | $-1.4\times10^{-4}$ |
| $\chi^2$ | 86 | 60 |
| $\nu=N-2$ | 30 | 30 |
| $Q$ | $2\times10^{-7}$ | 0.0008 |
| $R^2$ | 0.995 | 0.995 |
| Remarks | Goodness-of-fit: Very low | Goodness-of-fit: Hardly acceptable |

parameter model is given only for indicative purposes, since a 3-dimensional graph is normally required for the complete representation of $n$ as a function of $T_i^*$ and $G(T_i^*)^2$.

The results of least-squares and weighted least squares fitting for a 2-parameter model are presented briefly in Table 3. Table 4 contains the results concerning a 3-parameter model.

The problem of the evaluation of the candidate models is emphasized more clearly with the use of the well-known coefficient of regression $R^2$ in the table of results. This coefficient is often used as a criterion for the suitability of fitting.

Despite the fact that the above results concern the specific collector, and that the results differ from one collector to another, the following generally applicable remarks can be made, based on the analysis of a large number of test results conducted in our laboratory:

1. In some cases a 3-parameter model describes the collector behavior better than the 2-parameter model, especially for a collector with a black painted absorber. It is noteworthy that their difference is indicated not by coefficient of regression $R^2$, but by goodness-of-fit $Q$, which, in the case of the 2-parameter model was often below, or very close to, the acceptability limit. After all, $R^2$ indicates nothing about the quality of the model.
2. The acceptance criteria of the model parameters are almost always satisfied with the exception of $U_2$, the expanded uncertainty of which sometimes exceeds its own value. If this happens, the fitting should be reconsidered, otherwise there is a possibility of accepting negative values of parameter $U_2$. In this case it is preferable to repeat the fitting by considering the 2-parameters model.
3. The deviation of the model parameters values between LS and WLS is not particularly large in most cases. In spite of this, the quality of the fitting is improved significantly in the case of WLS.

## 5. CONCLUSIONS

We have developed a methodology for the evaluation of uncertainties in the testing results of solar collectors according to ISO 9806-1 and for the assessment of the performance of the models in use. This methodology is based on estimation of the experimental uncertainties and on the

Table 4. Results obtained using Least Squares (LS) and Weighted Least Squares (WLS) for a 3-parameter model

| Model: $n=n_0-U_1T_i^*-U_2G(T_i^*)^2$ | | |
|---|---|---|
| Quantity | Method of fitting | |
| | LS | WLS |
| $n_0$ | 0.6766 | 0.6768 |
| $U_1$ | 6.594 $K^{-1}$ W $m^{-2}$ | 6.527 $K^{-1}$ W $m^{-2}$ |
| $U_2$ | 0.0236 $K^{-2}$ $W^2$ $m^{-4}$ | 0.02418 $K^{-2}$ $W^2$ $m^{-4}$ |
| $\sigma_{n_0}$ | – | 0.00493 |
| $\sigma_{U_1}$ | – | 0.389 $K^{-1}$ W $m^{-2}$ |
| $\sigma_{U_2}$ | – | 0.0068 $K^{-2}$ W $m^{-2}$ |
| $\chi^2$ | 37 | 38 |
| $\nu=N-3$ | 29 | 29 |
| $Q$ | 0.11 | 0.12 |
| $R^2$ | 0.994 | 0.994 |
| Remarks | Goodness-of-fit: Acceptable | Goodness-of-fit: Acceptable |

implementation of the weighted least-squares fitting.

At a first stage it is necessary to determine all the experimental uncertainties following standardised rules of combination of all the sources of uncertainties, those which stem from sensor errors and those that express the instability of the measurements. After this, the fitting of the model to the experimental data is performed and its parameters are calculated, taking into account the uncertainties in the measurements and at the same time evaluating the quality of the fit of the model.

The basic conclusion is that a realistic assessment of the quality of the test results is not possible unless a check of the model suitability and of the whole testing procedure has previously been conducted. Implementation of the weighted least squares permits this check and at the same time enables the assessment of the possible expected error when the collector characteristic equation is used. It must also be stressed that if the goodness-of-fit is not acceptable, the combined standard uncertainties in model parameters will certainly not be reliable and the use of the model is hazardous.

The application of the above methodology in a large number of tests conducted in our laboratory showed that the expected expanded uncertainties in the predicted values of collector efficiency can be as much as 5%, provided that the requirements of the standard are satisfied.

Finally, it is noted that the proposed methodology is not limited to testing but it can be used also for the evaluation of a solar collector model of any kind.

## NOMENCLATURE

| | |
|---|---|
| $A_C$ | Collector aperture (m$^2$) |
| $c$ | Specific heat at mean temperature of water (J kg$^{-1}$ K$^{-1}$) |
| $G$ | Global incident solar irradiance (W m$^{-2}$) |
| $\dot{m}$ | Mass flowrate through the collector (kg s$^{-1}$) |
| $n$ | Collector efficiency $n = \dfrac{\dot{m}c(T_{out} - T_{in})}{A_C G}$ |
| $T_a$ | Ambient air temperature (°C) |
| $T_i^*$ | Reduced temperature difference $T_i^* = (T_{in} - T_a)/G$ (K W$^{-1}$ m$^2$) |
| $T_m^*$ | Reduced temperature difference $T_m^* = (T_m - T_a)/G$ (K W$^{-1}$ m$^2$) |
| $T_{in}$ | Temperature of water in collector inlet (°C) |
| $T_m$ | Mean temperature of water inside collector $T_m = (T_{in} + T_{out})/2$ (°C) |
| $T_{out}$ | Temperature of water in collector outlet (°C) |
| $\nu$ | Volume flowrate through the collector (m$^3$ s$^{-1}$) |
| $u_{A,q}$ | Type A standard uncertainties in an estimate $q$ |
| $u_{B,q}$ | Type B standard uncertainties in an estimate $q$ |
| $u_q$ | Standard uncertainties in an estimate $q$ |
| $\Delta T$ | Temperature difference $\Delta T = T_{out} - T_{in}$ (K) |
| $\rho$ | Density of water (kg m$^{-3}$) |
| $\sigma_q$ | Expanded uncertainty at a level of confidence of 95% in an estimate $q$ |

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