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Annex xx (Informative)
General guidelines for the assessment of uncertainty in solar collector efficiency testing

1. Introduction

The aim of this annex is to provide a general guidance for the assessment of uncertainty in the result of solar collector testing performed according to the present Standard. Testing laboratories are often invited to provide a statement of uncertainty in test results in quantitative tests, in the framework of their accreditation or of application of product certification schemes. It is not the aim of this annex to define whether and in which cases the calculation of uncertainty in test results is necessary.

This guidance concerns only in collector efficiency testing due to i) the great importance of the result of this testing for the user, and ii) the peculiarities of the calculations, since the final result of efficiency testing is not derived by a single measurement but by elaboration of a large number of primary measurements.

It is noted that the proposed methodology is one of the possible approaches for the assessment of uncertainty, and other approaches can be implemented. It is of the responsibility of each Laboratory to choose and to implement a scientifically valid approach for the determination of uncertainties, following the recommendations of the accreditation bodies, where appropriate. For a more detailed review of the different aspects of determination of uncertainties in solar collector testing see also [1, 2, 3, 4].

2. Measurement uncertainties in solar collector efficiency testing

The basic target of solar collector efficiency testing is the determination of the collector efficiency by measurements under specific conditions. More specifically, it is assumed that the behavior of the collector can be described by a *M*-parameter single node, steady state or quasi-dynamic model:

$$\eta=c_1p_1+c_2p_2+\ldots+c_Mp_M \qquad (1)$$

where:

η is the collector instantaneous efficiency.

$p_1, p_2, \ldots, p_N$ are quantities, the values of which are determined experimentally through testing

$c_1, c_2, \ldots, c_M$ are characteristic constants of the collector that are determined through testing.

In the case of the steady state model, for example, $M=3$, $c_1=\eta_0$, $c_2=U_1$, $c_3=U_2$, $p_1=1$, $p_2=(T_m-T_a)/G$ and $p_3=(T_m-T_a)^2/G$.

During the experimental phase, the output, solar energy and the basic climatic quantities are measured in *J* steady-state or quasi-dynamic state points, depending the model used. From these primary measurements the values of parameters $\eta, p_1, p_2, \ldots, p_M$ are derived for each point of observation j, $j=1\ldots J$. Generally, the experimental procedure of the testing leads to a formation of a group of *J* observations which comprise, for each one of the *J* testing points, the values of $\eta_j, p_{1,j}, p_{2,j}, \ldots, p_{M,j}$.

For the determination of uncertainties, it is necessary to calculate the respective combined standard uncertainties $u(\eta_j)$, $u(p_{1,j})$, $\ldots u(p_{M,j})$ in each observations point. It should be noted that in practice the

uncertainties $u(\eta_j)$, $u(p_{1,j})$, ...$u(p_{M,j})$ are almost never constant and the same for all points, but that each testing point has its own standard deviation.

For the calculation of the standard deviation (squared standard uncertainty) in each point *j*, the following general rules can be applied [5]:

I. Standard uncertainties in experimental data are determined by taking into account Type A and Type B uncertainties. According to the recommendation of ISO GUM [5], the former are the uncertainties determined by statistical means while the latter are determined by other means.

II. The uncertainty $u(s)$ associated with a measurement s is the result of a combination of the Type B uncertainty $u_B(s)$, which is a characteristic feature of the calibration setup, and of the Type A uncertainty $u_A(s)$, which represents fluctuation during sampling of data. If there is more than one independent source of uncertainty (Type B or type A) u_k, the final uncertainty is calculated according to the general law of uncertainties combination:

$$u = \left(\sum_k u_k^2 \right)^{1/2} \tag{2}$$

III. Type B uncertainty $u_B(s)$ derives from a combination of uncertainties over the whole measurement chain, taking into account all available data, such as sensor uncertainty, data logger uncertainty, uncertainty resulting from the possible differences between the measurand values perceived by the measuring device. Relevant information has to be obtained from calibration certificates or other technical data related to the devices used.

IV. By nature, Type A uncertainties depend on the specific conditions of measurement and they account for the fluctuations in the measured quantities during the measurement. Type A uncertainty $u_A(s)$ derives from the statistical analysis of experimental data. In some cases (for example in the case of the steady-state model), the best estimate of S is the arithmetic means s of the I repeated observations s_i ($i=1...I$) and its Type A uncertainty is the standard deviations of the mean:

$$s = \frac{\sum_{i=1}^{I} s_i}{I}, \text{ and } u_A(s) = \left(\frac{\sum_{i=1}^{I} (s_i - s)^2}{I(I-1)} \right)^{1/2} \tag{3}$$

In some other cases (for example in the case of the quasi-dynamic model where no arithmetic mean of the repetitive measurements is used) uncertainty $u_A(s)$ can be equal to zero.

V. The term *combined standard uncertainty* means the standard uncertainty in a result when that result is obtained from the values of a number of other quantities. In most cases a measurand Y is determined indirectly from P other directly measured quantities X_1, X_2, ...X_P through a functional relationship $Y=f(X_1, X_2, ...X_P)$. The standard uncertainty in the estimate y is given by the *law of error propagation*:

$$u(y) = \left(\sum_{i=1}^{P} \left(\frac{\partial f}{\partial x_i} \right)^2 (u(x_i))^2 + 2 \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \operatorname{cov}(x_i, x_j) \right)^{1/2} \tag{4}$$

An example of such indirect determination in the case of solar collector efficiency testing is the determination of instantaneous efficiency η, which derives from the values of global solar irradiance in the collector level G, fluid mass flow rate m, temperate difference ΔT, collector area A and specific heat capacity c_f. Thus, in this case the standard uncertainty $u(\eta_j)$ in each value η_j of instantaneous efficiency is calculated by the combination of standard uncertainties in the values of the primary measured quantities, taking into account their relation to the derived quantity η.

3. Fitting and uncertainties in efficiency testing results

During analyzing the data a least square fitting of the model equation is performed, in order to the determine the values of coefficients $c_1, c_2,...,c_M$ for which the model of equation (1) represents the series of J observations with the greatest accuracy.

Since in reality, the typical deviation is almost never constant and the same for all observations, but that each data point $(\eta_j, p_{1,j}, p_{2,j},...,p_{M,j})$ has its own standard deviation σ_j, an interesting solution is the use of the ***weighted least square (WLS)*** method, which calculates, on the base of the measured values and their uncertainties, not only the model parameters but also their uncertainty. In the case of WLS, the maximum likelihood estimate of the model parameters is obtained by minimising the chi-square function:

$$\chi^2 = \sum_{j=1}^{J} \frac{\left(\eta_j - (c_1 p_{1,j} + c_2 p_{2,j} + ... c_N p_{M,j})\right)^2}{u_j^2} \tag{5}$$

where u_j^2 is the variance of the difference $\eta_j - (c_1 p_{1,j} + c_2 p_{2,j} + ... c_N p_{M,j})$:

$$u_j^2 = Var\left(\eta_j - (c_1 p_{1,j} + c_2 p_{2,j} + ... c_N p_{M,j})\right) = \left(\boldsymbol{u}(\eta_j)\right)^2 + c_1^2 \left(u(p_{1,j})\right)^2 + ... + c_M^2 \left(u(p_{M,j})\right)^2 \tag{6}$$

Finding coefficients $c_1, c_2,...,c_M$ and their standard uncertainties by minimizing chi-square function is complicated, because of the non-linearity present in equation (5). A strategy is therefore to find these uncertainties numerically. A method for the case of a M-parameter model is presented below [6].

Let K be a matrix whose JxM components $k_{j,m}$ are constructed from M basic functions evaluated at the J experimental values of $p_1, ...,p_M$ weighted by the uncertainty u_j :

$$\boldsymbol{k_{j,m}} = \frac{\boldsymbol{p_{m,j}}}{\boldsymbol{u_j}}, \qquad \boldsymbol{K} = \begin{vmatrix} \frac{\boldsymbol{p_{1,1}}}{\boldsymbol{u_1}} & . & . & \frac{\boldsymbol{p_{1,M}}}{\boldsymbol{u_1}} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \frac{p_{1,J}}{\boldsymbol{u_J}} & . & . & \frac{p_{M,J}}{\boldsymbol{u_J}} \end{vmatrix} \tag{7}$$

Let also L be a vector of length J whose components l_j are constructed from values of η_j to be fitted, weighted by the uncertainty u_j:

$$l_j = \frac{\eta_j}{u_j}, \qquad \boldsymbol{L} = \begin{vmatrix} \eta_1 / \boldsymbol{u}_1 \\ . \\ . \\ . \\ \eta_J / \boldsymbol{u}_J \end{vmatrix} \tag{8}$$

The *normal equation of the least square problem* can be written:

$$(K^T \bullet K) \bullet INV(C) = K^T \bullet L \quad (9)$$

where C is a vector whose elements are the fitted coefficients.

Given the fact that for the calculation of variances u_j^2 the knowledge of coefficients $c_1, c_2,...,c_M$ is needed, a possible solution is to use the values of coefficients calculated by standard least squares fitting as the initial values. These initial values can be used in equation (6) for the calculation of u_j^2, $J=1...J$ and the formation of matrix K and of vector L.

The solution of equation (9) gives the new values of coefficients $c_1, c_2,...,c_M$, which however are not expected to differ noticeably from those calculated by standard least squares fitting and used as initial values for the calculation of u_j^2.

Moreover, $Z=INV(K^T \bullet K)$ is a matrix whose diagonal elements $z_{k,k}$ are the squared uncertainties (variances) and the off-diagonal elements $z_{k,l}= z_{l,k}$, $k \neq l$ are the covariance between fitted coefficients:

$$u(c_m) = \sqrt{z_{m,m}}, \; m=1,...,M \quad (10)$$

$$Cov(c_k,c_l)= z_{k,l}= z_{l,k}, \; k=1,...,M \text{ and } l=1,...,M \text{ and } k \neq l \quad (11)$$

It should be noted that the knowledge of covariance between the fitted coefficients is necessary if one wishes to calculate, in a next stage, the uncertainty $u(\eta)$ in the predicted values of η using equations (1) and (4).

Equation (9) can be solved by a standard numerical method, for example, by Gauss-Jordan elimination. It is also possible to use matrix manipulation functions of commonly used spreadsheet software.

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